Simulation of wave loads and dynamic stresses in ship hulls in irregular waves

Jan Jankowski
Polish Register of Shipping, Poland, mailbox@prs.pl

Marian Bogdaniuk
Polish Register of Shipping, Poland, m.bogdaniuk@prs.pl

Abstract

The accuracy of predicting wave generated stresses in ship structure is still unsatisfactory. This has an impact on safety at sea. Therefore, methods based on simulation of ship motion in waves, including the simulation of ship structure response to waves, green sea and sloshing, are being developed. The simulation results are transformed into probability distributions used to determine the safety level corresponding to serious damage of ship hull or hatch cover structure leading to sinking of the ship.

The paper presents the method of wave loads and stress prediction aimed at determining class rule requirements.

Keywords

simulation of ship motion; ship hull structure; wave loads; wave generated stresses.

Nomenclature

\( a_{vi} \) values of vertical accelerations in selected points of the inner bottom,
\( D \) rotation matrix,
\( D_f \) fatigue damage parameter,
\( D_\Omega \) matrix which transforms Euler components of rotational velocity \((\dot{\varphi}, \dot{\theta}, \dot{\psi})\) into \(\Omega\),
\( f \) Weibull probability density function,
\( F \) Weibull cumulative probability function,
\( F_A \) and \( M_{QA} \) additional forces and moments such as damping forces or those generated by the rudder,
\( F_c \) transverse force acting on the bulkhead single corrugation,
\( F_W, F_D \) and \( F_R \) Froude–Krylov, diffraction and radiation forces, respectively,
\( G = (0, 0, -g) \) gravitational acceleration vector,
\( I \) number of stress range intervals,
\( I_1 \) number of selected points in the bottom and side plating below hopper tank tops,
\( I_2 \) number of selected points on the line of lower ends of the frames,
\( I_3 \) number of selected points on the inner bottom plating where the vertical acceleration is determined,
\( I_f \) number of stress range intervals taken into account in numerical representation of function \(f\),
\( I_s \) number of simulations,
\( K, m \) parameters of S-N curve,
\( L = (l_{Q1}, l_{Q2}, l_{Q3}) \) ship angular momentum,
\( m \) ship mass,
\( M_{QW}, M_{QD}, M_{QR} \) moments of forces \(F_W, F_D\) and \(F_R\) in relation to the mass centre,
\( n_i, i=1,...,I \) number of stress cycles in interval \(i\),
\( N_i, i=1,...,I \) number of the stress cycles \(\Delta \sigma_i\), causing the fatigue damage,
\( p_{1i} \) values of water pressure in selected points of the hull plating below hopper tanks tops,
\( p_{2i} \) as above at the level of hopper tanks tops,
\( R_{UQ}=(x_{UQ1}, x_{UQ2}, x_{UQ3}) \) position vector of the ship mass centre in relation to the inertial system \(U\), moving with a constant speed equal to the average speed of the vessel,
\( t_s \) time of simulation, \(i=1,...,I_s\),
\( V_Q = (V_{Q1}, V_{Q2}, V_{Q3}) \) velocity of the ship mass centre,
\( W_{1i}, W_{2i}, W_{3i} \) pressure influence coefficients,
\( \Delta \sigma_i \) stress range in interval \((\Delta \vec{\sigma},\Delta \vec{\sigma})\), \(i=1,...,I\),
\( \Delta \vec{\sigma} \) average stress range in \(i\)-th interval, \(i=1,...,I\).
\( \eta, \xi, \varepsilon \) parameters of the Weibull distribution,
\( (\varphi, \theta, \psi) \) Euler’s angles representing roll, pitch, yaw,
\( \sigma \) stress,
\( \Omega = (\omega_1, \omega_2, \omega_3) \) ship angular velocity.

1. Introduction

The prediction of wave generated stress level in the structure is of key importance in ship hull safety assessment. The stresses can cause failure of the structure in the form of plastic flow, buckling of some structure members or fatigue cracks. The accuracy of wave stresses prediction is still unsatisfactory and therefore continuous development of methods to predict stresses in ships hull structures is necessary.

Severe weather conditions, which can randomly occur during the ship life, have a major impact on sustaining ship structure safety. The “extreme load cases” depend also on the dimensions, shape, mass distribution and speed of the ship. To determine the “extreme load cases” all possible sea states – defined by significant wave height and average zero up-crossing period, as well as all possible ship headings should be analysed. The computation takes into account their numerical representation, corresponding to the scatter diagrams, which determine the probabilities of sea state occurrence (Hogben, 1986).

The classic method for predicting dynamic stresses in hull structures caused by waves is based on spectral analysis of wave loads with stresses in the structure calculated using FEM models. However, it is impossible to predict stresses in the structure accurately using the spectral analysis as the phase shift between, for example, wave generated pressures and ship hull accelerations at all parts of the hull structure are lost in this method. Also the nonlinear effects such as bow flare effect, green water, etc. are disregarded in spectral analysis (eg. Guedes Soares, 1999).

The paper presents the method for predicting stresses in ship structure. According to this method equations of ship motion in irregular waves are solved numerically in time domain taking into account nonlinear effects. Stresses at any point of hull structure can be calculated in time domain using the concept of influence coefficients of wave loads.

A method of simulating sloshing loads in flooded holds, induced by the ship moving in waves (Warmowska, Jankowski, 2006), and the prediction of green seas loading on bulk carrier hatch covers (over first hold) have also been developed (Vassalos et al., 2003), (Jankowski, Laskowski, 2005).

The results obtained using spectral analysis (frequency domain) or simulation (time domain) can be transferred into the probabilistic domain. The paper presents the results of simulations of the wave bending moment, stresses in side frames, stresses in hatch covers, sloshing loads on corrugated bulkhead, generated by irregular waves in a panamax bulk carrier hull structure and their transformation into probability distribution functions. The assessment of fatigue life is also presented.

The method for prediction of wave loads and stresses in ship hull structures is used to determine design load cases for PFS classification rules.

2. Simulation of ship motion in irregular waves

The simulation of vessel motions in waves is based on numerical solutions of non-linear equations of motion (1). The non-linear model used is presented in (Jankowski, 2006).

It is assumed that the hydrodynamic forces acting on the ship and defining the equations of its motions can be split into Froude-Krylov forces, diffraction and radiation forces as well as other forces, such as rudder forces and non linear damping.

The Froude-Krylov forces are obtained by integrating the pressure caused by irregular waves, undisturbed by the presence of the ship, over the actual wetted ship surface.

The diffraction forces are determined as a superposition of diffraction forces caused by the harmonic components of the irregular wave. The irregular wave is assumed to be a superposition of harmonic waves. It is assumed that the ship diffracting the waves is in its mean position. This is possible under the assumption that the diffraction phenomenon is described by a linear hydrodynamic problem. The variables of diffraction function are given as the product of space and time variables with the space factor of the function being the solution of the hydrodynamic problem and the known time factor. Such an approach significantly simplifies calculations because bulky computations can be performed at the beginning of the simulations and the ready solutions can be applied for determining the diffraction forces during the simulation.

The radiation forces are determined by added masses for infinite frequency and by the so-called memory functions given in the form of convolution. The memory functions take into account the disturbance of water, caused by the preceding ship motions, affecting the motion of the ship in the time instant considered, (Cummins, 1962).

The equations of ship motion in irregular waves are written in the non-inertial reference system. The system is fixed to the ship in the centre of its mass Q and the equations of ship motion assume the following form (Jankowski, 2006):

\[
m[V_{\Omega}(t) + \Omega(t) \times V_{\Omega}(t)] = F_w(t) + F_D(t) + F_R(t) + F_T(t) + F_A(t) + mD^{-1}G, \\
L(t) + \Omega(t) \times L(t) = M_{QW}(t) + M_{QD}(t) + M_{QR}(t) + M_{QT}(t) + M_{QA}(t), \\
R_{UQ}(t) = V_{\Omega}(t) - \Omega(t) \times R_{UQ}(t), \\
(\varphi(t), \theta(t), \psi(t))^T = D^{-1} \Omega(t)
\]
The ways of solving 3D hydrodynamic problems and determining forces appearing in the equation of motion are presented in (Jankowski, 2006). The non-linear equations of motion are solved numerically according to the method presented in (Rolston, 1975).

3. Simulation of ship structure response to irregular waves

3.1. Wave generated bending moment

Wave vertical bending moments, generated in hull cross section \( x = x_1 \), are calculated as the result of action of external pressures excited by waves and inertial loads acting on aft part of the hull (for \( x \leq x_1 \)). An example of the time history of vertical bending moment \( M_v(x_1,t) \) in the midship of a panamax bulk carrier is presented in Fig. 1. The presented moment is the superposition of still water bending moment \( M_s \) and wave bending moment \( M_w \).

![Fig.1: Time history of vertical bending moment \( M_v \) in the midship of hull](image)

3.2 Wave generated stresses in side frames

The characteristic feature of single side skin bulk carriers is strong dependence of stress values in their side frames not only on values on external pressures acting on sides but also on pressures of water acting on bottom plating and pressures of cargo acting on inner bottom and hopper tanks plating (Fig. 2).

The pressures are the sum of static and dynamic components. Wave pressures of water and inertia components of cargo pressure due to ship accelerations are the dynamic components.

Stress values in an individual frame depend on the pressures acting on quite long part of hull module containing the frame.

The stress values in frames are approximately calculated using the concept of influence coefficients of pressure values in some selected points of the hull module.

![Fig. 2: Deformation of cross section of bulk carrier in empty hold](image)
a) selected points

b) interpolation of water pressure on side, below the top of hopper tank

c) interpolation of water pressure on bottom

d) water pressure on side, above hopper tank

Fig. 3: Approximation of pressure on the side and bottom

Type „A“ — a primitive separated bracket
Type „B“ — improved shape of the bracket
Type „C“ — bracket integrated with the frame

Fig. 4: Different designs of analysed frames

These pressures are calculated as the result of simulation of ship response to waves.

Linear interpolation of the pressure values is applied to calculate the pressure at any point of the structure.

An example of the selection and interpolation is shown in Fig. 3.

The stress value at a point of selected side frame is calculated according to the formula:
\[ \sigma = \sum_{j=1}^{t_1} p_{1j} W_{1j} + \sum_{j=1}^{t_2} p_{2j} W_{2j} + \sum_{j=1}^{t_3} a_{3j} W_{3j} \] (2)

The influence coefficients values \( W_{1j}, W_{2j}, \) and \( W_{3j} \) are calculated using an FEM model of a ship hull module, using shell, beam and rod finite elements.

Values of the influence coefficients are to be known before performing the simulation of ship dynamics in waves. Three possible kinds of ship frames design were considered (Fig. 4).

Stresses \( \sigma \) calculated according to formula (2), were used to assess fatigue life values of the frames at their lower ends, in places shown in Fig. 4.

\[ \sigma[MPa] \]
\[ t[s] \]

**3.3. Sloshing forces acting on the corrugated bulkhead**

It is assumed that the loss of bulkhead integrity and the 'progressive flooding' follow loss of ultimate strength capacity of the corrugated part of the bulkhead, bent by sloshing in flooded hold. The sloshing loads on a single corrugation of the bulkhead is determined during the simulation of ship motion in waves.

The moving ship causes the motion of water in flooded hold, which induces the pressure on the corrugation. The 2-D sloshing model (Warmowska, 2006) was used to simulate water motion in the plane along longitudinal axis of the ship (Fig. 6).

\[ \frac{V}{V_t} \] velocity of water particles

As the measure of corrugated bulkhead ultimate capacity, the ultimate transverse force \( F_u \) value of single corrugation corresponding to linear distribution of pressure along height of the bulkhead is used. \( F_u \) value is calculated before simulation of sloshing, applying a non linear FEM model of corrugation. Therefore, force \( F_i \) acting on the corrugation, being the result of the pressure integration over the actual wetted part of the corrugation, is simulated (an example for the panamax bulkcarrier considered is presented in Fig. 7).

**3.4. The stresses generated in the hatch cover by green seas**

The following theoretical model determining green seas loading on hatch covers was assumed (Jankowski, Laskowski, 2005):

- The green seas occurrence is identified in the simulation of ship motion in irregular waves.
- Green seas on foredeck usually occur when the ship meets a series of three or more high waves. The first wave normally "collapses on the foredeck in the form of horizontal jests of water travelling aft and impinging horizontal and vertical impacts on the front coamings and covers respectively" (Vassalos, 2003). This kind of load is not taken into account in the present model as it does not influence the hatch cover strength. In the second wave, the forepart of the ship submerges due to the bow pitching down. In this case it is assumed that the hatch cover load can be approximated by the pressure in incident waves acting on the cover level, and by the acceleration of the ship in way of the hatch cover and by change of the amount of water mass over the cover.
- The phenomenon of green seas occurs only in severe wave conditions when the ship
normally faces the waves and the speed is reduced.

The stresses in hold No 1 hatch cover caused by the green seas determined during simulations of ship motion in irregular waves, were computed using a linear elastic FEM of the cover structure and applying the concept of influence coefficients for values of pressure acting on the cover, similar to the concept applied to side frames. Then, these elastic stress values were approximately transferred into ultimate capacity of the cover structure. An example of results for the panamax bulk carrier considered is shown in Fig. 8.

4. Statistical distribution of ship structure response to waves

Simulation of ship structure response to irregular waves in an assumed sea state enables the determination of its local extrema and the number of the extrema. Every simulation lasted 30 000s. The numerical probability density function of the considered ship structure response can be obtained by multiplying the number of structure response extrema belonging to the particular interval by probability of the given sea state occurrence and dividing it by the total number of extrema occurrences in the sea state and by the length of the interval and then, summing them for all sea states. The sea states (irregular waves) are determined by a scatter diagram, representing the probabilities of particular sea states occurrence (Hoyben et al, 1986). Different scatter diagrams, representing different ship routes, have been taken into account (Jankowski, Bogdaniuk, 2007). The numerical probability density is normally approximated by Weibull distribution. The cumulative probability function takes the form:

\[ F(x) = 1 - \exp \left[ -\left( \frac{x - \xi}{\eta - \xi} \right)^\zeta \right], \quad (3) \]

with the probability density function

\[ f(x) = \frac{\xi}{\eta - \xi} \left( \frac{x - \xi}{\eta - \xi} \right)^{\zeta-1} \exp \left[ -\left( \frac{x - \xi}{\eta - \xi} \right)^\zeta \right], \quad (4) \]

where \( \eta, \xi \) and \( \zeta \) are the Weibull distribution parameters.

Basing on the simulation of the wave vertical bending moment - see Fig. 1, the statistical and probability distribution (approximation by Weibull distribution) of the extrema for:

- maxima above still water bending moment, and
- minima below the still bending moment

are presented in Fig 9 and 10 respectively for the panamax bulk carrier considered.

![Fig. 8: Time history of stresses σ in the hatch cover girder flange caused by green seas](Image)

![Fig. 9: Numerical and Weibull probability distribution of wave bending moment maxima](Image)

![Fig. 10: Numerical and Weibull probability distribution of wave bending moment minima](Image)
The statistical and probability distribution of the stresses in frames refer to the minima of simulated stresses. The distribution for the designs of the frames shown in Fig 4, Type A and C, are presented in Fig 11 and 12. The frames are in the middle of hold No 4.

The simulation of sloshing was carried out in hold No 4 and the sloshing force was calculated for the bulkhead between hold No 4 and 5.

The distribution of the sloshing force $F_c$ acting on the single bulkhead corrugation is the distribution of the force maxima obtained in the simulation of water motion in flooded hold (Fig. 7). The distribution is presented in Fig. 13.

The distribution of the stresses in the hold No 1 hatch cover side girder generated by green seas is the distribution of the maxima of the stress sides shown in Fig. 8. The distribution is presented in Fig. 14.

The Weibull distribution enables calculation of the characteristic value $M$ of random variable considered. This value is determined by the equation:

$$1 - F(M) = \frac{1}{N},$$  \hspace{1cm} (5)

where $F$ is the Weibul distribution (3), $N$ is the number of cycles of random variable in the ship’s lifetime ($N=10^8$). For Weibull distribution the characteristic value can be calculated from the following formula:

$$M = \theta + \exp[\ln(-\ln(1/N)) / \xi + \ln(\eta - \theta)]$$  \hspace{1cm} (6)

The characteristic values of the wave bending moments and stresses in frames are presented in Table 1.
Table 1: Characteristic values of wave bending moment and stresses in the frames (Fig. 4)

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Weibull parameters</th>
<th>Characteristic value acc. to (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{M} v$ [kNm]</td>
<td>$\xi$</td>
<td>$\eta$</td>
</tr>
<tr>
<td>$\bar{M} v$ [kNm]</td>
<td>1.26</td>
<td>$2.9 \times 10^5$</td>
</tr>
<tr>
<td>$\sigma_c$ as built [Mpa]</td>
<td>1.06</td>
<td>245</td>
</tr>
<tr>
<td>$\sigma_c$ increased [Mpa]</td>
<td>0.90</td>
<td>121</td>
</tr>
</tbody>
</table>

In the Table the following denotations are used:

$\bar{M} v$ - is the random variable of the wave bending moment maxima (sagging).

$\bar{M} v$ - is the random variable of the wave bending moment minima (hogging).

$\sigma_c$ as built is the stress at frame point denoted in Fig.4.

$\sigma_c$ increased is the stress at frame point denoted in Fig.4; the section modulus of this frame is increased 2 times in relation to “as built”.

In case of sloshing and green seas generating stresses in bulkhead and hatch cover the approximation of the numerical probability distribution by Weibull function is not satisfactory (see Fig 15 and Fig 14). Therefore:

- the numerical distribution based on a much longer simulation time, or
- other probability distribution function to approximate the numerical distribution

should be used to assess the strength of bulkhead and hatch cover.

5. Fatigue strength of the frames

The irregular wave generated stress ranges $\Delta \sigma$ (Fig. 15) can be grouped in the intervals $(\Delta \bar{\sigma}_i, \Delta \bar{\sigma}_{i+1})$. As the result of simulation of the ship motion (and the stresses in the frames) in different waves, determined by scatter diagrams giving the probability of sea states occurrence, and for different ship headings in relation to wave directions, the number $n_i$ of the stress ranges $\Delta \sigma_i$ in each interval, $i=1...I$, is determined. The numerical distribution is obtained by dividing the number $n_i$, $i=1...I$, by the total number of stress range occurrences and by the length of the interval. Approximation of the ship function by Weibull distribution is also presented in Fig.16 and 17.

According to Palgrem-Miner hypothesis the fatigue damage caused by stress ranges $\Delta \bar{\sigma}_i = (\Delta \bar{\sigma}_i + \Delta \bar{\sigma}_{i+1})/2$ is accumulated in the frames.

The fatigue damage parameter $D_i$ of the frame can be approximated by the following formula:

$$D_i = \sum_{i=1}^{I} \frac{n_i}{N_i}, \quad (7)$$

where $N_i$, $i=1...I$, is the number of cycles of the average stress range $\Delta \bar{\sigma}_i$, causing the fatigue damage. The damage will appear when $D_i > 1$. Numbers $N_i$, $i=1...I$, can be determined by the following formula:

$$N_i = \frac{K}{\Delta \sigma_i^m}, \quad (8)$$

where, $K$ and $m$ are the parameters of the S-N curve (UK Department of Energy, 1990).
Substituting (8) into (7) the following formula is obtained:

$$D_j = \frac{1}{K} \sum_{i=1}^{I_s} n_i \Delta \sigma_i, \quad (9)$$

If the Weibull probability density function (4), approximating the numerical distribution of $\Delta \sigma_i$ is used, formula (9) takes the form:

$$D_j = \frac{N_o}{K} \sum_{i=1}^{I_s} \Delta \sigma_i f(\Delta \sigma_i)(\Delta \sigma_i - \Delta \sigma), \quad (10)$$

where:

$N_o$ is the average number of stress range cycles computed with the use of the following formula:

$$N_o = \left( \sum_{i=1}^{I_s} \frac{n_i}{T_{i}} \right) I_{t}, \quad (11)$$

$I_{t}$ is the number of ship motion simulations, $t_{ij}, j=1,...,I_s$, is the time of simulation $i=1,...,I$, $T_{i}=7.88 \times 10^8$ s, is the design life corresponding to 25 years.

$I_f$ is the number of stress range intervals taken into account in numerical representation of the probability density function $f$.

The number of years $T_f$ after which fatigue cracks are expected is determined by the formula:

$$T_f = \frac{25}{D_f} \quad (12)$$

The fatigue damage parameters determined according to formula (8) and (10) and for three frame designs considered (Fig.4) is presented in Table 2.

<table>
<thead>
<tr>
<th>Design type</th>
<th>According to formula no</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(9) &amp; (12)</td>
</tr>
<tr>
<td>D</td>
<td>$T_f$ (years)</td>
</tr>
<tr>
<td>A</td>
<td>2.05</td>
</tr>
<tr>
<td>B</td>
<td>1.21</td>
</tr>
<tr>
<td>C</td>
<td>0.021</td>
</tr>
</tbody>
</table>

The fatigue damage parameters were also determined for the frames with increased section modulus 2 times. The results are presented in Fig. 18 and Table 3.

<table>
<thead>
<tr>
<th>Design type</th>
<th>According to formula no</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(9) &amp; (12)</td>
</tr>
<tr>
<td>D</td>
<td>$T_f$ (years)</td>
</tr>
<tr>
<td>A</td>
<td>0.2380</td>
</tr>
<tr>
<td>B</td>
<td>0.1136</td>
</tr>
<tr>
<td>C</td>
<td>0.0098</td>
</tr>
</tbody>
</table>

6. Conclusions

The accuracy of predicting wave generated stresses in ship structure is still unsatisfactory. This is mainly due to the application of the spectral analysis (frequency domain) based on linear methods. Therefore, the nonlinear methods should be applied to predict the wave generated loads and the stresses in the ship structure. The simulation of ship response to waves (time domain) enables determination of wave bending stresses in the side frames, stresses in the hatch covers caused by green seas, sloshing load on corrugated bulkheads in flooded hold and then to transfer the results into the probability domain.
The results of computations show that:

1. The simulation of ship structure response to irregular waves is a method which describes the physical phenomenon directly (Fig. 1, 5, 7 and 8) – providing results similar to those obtained in the experiment.

2. There is no problem in transferring the results of the simulation of ship structure response to waves into the probabilistic domain.

3. The approximation of numerical probability density function by Weibull probability distribution show that the approximation is good for bending moments and stresses in the side frames but for sloshing and stresses in the hatch cover the approximation is not satisfactory. For determining the safety level of corrugated bulkhead and hatch cover in these cases the numerical probability distribution should be based on a much longer time of simulation.

4. The characteristic value of vertical wave bending moment determined in the present method (Table 1) for a panamax size bulk carrier is greater by 18% than that determined by IACS UR S11 (2001), giving for the analysed bulkcarrier the following values:

   \[ M_{v}^\text{−} = 2.7 \times 10^6 \text{ kNm (sagging)}, \text{ and} \]
   \[ M_{v}^\text{+} = 2.5 \times 10^6 \text{ kNm, (hogging), respectively.} \]

   If we take into account the fact that the wave bending moments determined by IACS UR S11 correspond to the North Atlantic wave conditions whereas the present methods – to the World Wide trading, the difference is greater (see also Guedes, Soares, 1996).

   The ratio between \( M_{v}^\text{−} \) and \( M_{v}^\text{+} \) bending moment determined by the present method (for all possible sea states and heading) is equal to 1.1, while the ratio determined by IACS is equal to 1.08.

5. The characteristic value of the stresses in the as built side frames is much greater than the yielding point of the frame material (Table 1.). The ship considered sank in heavy weather conditions seven years ago due to the loss of side integrity, probably followed by collapse of the bulkhead due to sloshing and progressive flooding. Doubling the section modulus of the frames in the lower part significantly increases the safety of the ship (Table 1).

6. The fatigue strength of the as built frame type C (Fig. 4), with the integrated bracket, is satisfactory (Table 2.). This type of frame designs should be applied, as the other solutions (type A and B - Fig. 4) are unsatisfactory.

The method presented in the paper used to determine the long term probability distributions of the ship wave loads and stresses in the structure, can easily be adopted to the risk models based on the fault tree used to determine the safety level of ship structure (Jankowski, Bogdaniuk, 2007).

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