

Wave load cases for scantlings of bulk carrier structures

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ABSTRACT: The accuracy of predicting wave-generated stresses in ship structure continues to remain unsatisfactory, affecting safety at sea. Therefore, methods based on simulation of ship motion in waves are being developed, including the simulation of ship structure response to waves, green seas and sloshing. The simulation results are transformed into probability distributions used to determine stresses with assumed probability of their occurrence. The paper presents a method of wave generated stress prediction based on simulation of ship motion, loads and stresses in ship structure in waves.

1 INTRODUCTION

Presently, classification societies require the middle part of a ship structure to be assessed using the Finite Element Method (FEM). This method is well established as it gives satisfactory results of structure response (stress, deflection) to the loads applied. This implies that apposite assessment of ship structure depends on the accuracy of loads applied, mainly the wave loads.

Waving of the sea is a stochastic process and various realizations of the processes (irregular waves) and ship headings to the waves, which can occur with a certain probability in a ship's life, should be taken into account for the assessment of ship structures. This approach determines long-term response of ship structures to waves.

The prediction of wave generated stress level in the structure is of key importance in ship hull safety assessment. The stresses can lead to structure failure in the form of plastic flow, buckling of some structure members or fatigue cracks. The accuracy of wave stresses prediction is still unsatisfactory and therefore continuous development of methods to predict stresses in ship hull structures is necessary.

Severe weather conditions, which randomly occur during a ship's life, have a major impact on sustaining ship structure safety. The "extreme load cases" also depend on ship dimensions, her shape, mass distribution and ship speed. All possible sea states – defined by significant wave height and average zero up-crossing period, as well as all possible ship headings should be analyzed to determine the "extreme load cases". The computations take into account the representation of sea states, corresponding to the scatter diagrams,

which determine the probabilities of sea state occurrence (IACS Rec. No. 34).

The classic method for predicting dynamic stresses in hull structures caused by waves is based on a spectral analysis of wave loads. However, this method is based on linear models and the nonlinear effects such as bow flare effect, green water, etc. are disregarded in this method (eg. Guedes Soares, 2000). Usually the phase shift between, for example, wave generated bending moments, pressures and ship hull accelerations are lost in spectral analysis, which makes it impossible to predict stresses in the structure accurately.

The paper presents a method for predicting stresses in ship structure. According to this method equations of ship motion in irregular waves are solved numerically in time domain taking into account nonlinear effects. Stresses at any point of the hull structure can be computed in time domain using the concept of influence coefficients of wave loads (Jankowski, Bogdaniuk 2007).

However, such an approach is applicable in R&D projects but not in design practice. Therefore, simplifications should be introduced. Studies indicate that long-term response of the structures to sea waving can be approximated by appropriately chosen short-term response (response to one irregular wave). In such approximations the short-term response varies for different types of ship structures.

This paper presents a method of identification of wave load cases – irregular waves, determined by wave parameters and the ship heading, that have a dominant impact on the response of selected bulk carrier structures.

2 PREDICTION OF SHIP STRUCTURE RESPONSE TO WAVES

Seas and oceans are normally divided into distinct areas A_l , $l = 1, 2, \dots, n$, (Hogben et al, 1986), characterized by the spectral density function. Wave spectrum representing the steady state sea conditions (short-term sea state) depends on the significant wave height H_s and average zero up-crossing period T_o (Ochi, 1998).

The short-term response of the ship to waves (e.g. wave bending moment M_W) is a set of probability distributions (e.g. probability density functions) of the given random variables for one sea state in a given area A_l , for various ship courses etc.

Long-term statistics of the given ship response is the accumulation of response statistics referring to: sea areas A_l , $l=1, \dots, n$, short-term sea states, ship courses in relation to waves and ship's loading conditions, taking into account the frequencies of their occurrence.

The long-term probability density function $f(y)$ of the ship response y (e.g. wave bending moment M_v as the random variable) can be expressed as:

$$f(y) = \sum_m \sum_l \sum_k \left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{klm}(y | (H_s, T_o)) g(H_s, T_o) dH_s dT_o \right) p_{kl} p_l p_m \quad (1)$$

where $f(y | (H_s, T_o)) =$ probability density function of the random variable y in the sea state condition (H_s, T_o) and $g(H_s, T_o)$ is probability density function of of sea state occurrence.

Taking into account the formula determining the conditional distribution, and by approximating (by the relevant sums) the integral occurring in (1), the following formula is obtained:

$$f(y) = \sum_m \sum_l \sum_k \sum_j \sum_i f_{ijklm}(y | (H_s, T_o)) p_{ijl} p_{kl} p_l p_m \quad (2)$$

where $f_{ijklm} =$ the short term probability density function of random variable y ; $p_m =$ the probability of the ship's loading condition occurrence (different drafts for different loading conditions); $p_l =$ the probability of ship presence in sea area A_l ; $p_{kl} =$ the probability of ship course in relation to waves in sea area A_l (uniform distribution in the interval $[0, 2\pi]$ is used); $p_{ijl} =$ the probability of the short-term sea state, determined by (H_s, T_o) , occurrence in the sea area A_l , $l=1, \dots, n$.

The probability distributions of the sea states occurrence are given in the form of a matrix – called scatter diagram, which presents the probabilities p_{ijl} of sea state occurrence in the interval $[H_{si}, H_{si+1}]_l \times$

$[T_{oj}, T_{oj+1}]_b$, $i = 1, \dots, r$, $j = 1, \dots, s$, $l=1, \dots, n$, (Hogben et al, 1986).

The numerical long-term probability density functions (2) of random variable y , representing different ship structure response to waves, is computed, basing on simulations of ship motion and the structure's behavior in irregular waves.

The shipmaster's decisions exercising good seamanship like e.g. speed reduction in high seas and change of course in relation to the waves are projected in the long-term procedure.

3 SIMULATION OF SHIP MOTION IN IRREGULAR WAVES

The simulation of vessel motions in waves is based on numerical solutions of non-linear equations of motion. The non-linear model used is presented in (Jankowski, 2006).

It is assumed that the hydrodynamic forces acting on the ship and defining the equations of its motions can be split into Froude-Krylov forces, diffraction and radiation forces as well as other forces, such as rudder forces and non linear damping.

The Froude-Krylov forces are obtained by integrating the pressure caused by irregular waves, undisturbed by the presence of the ship, over the actual wetted ship surface.

The diffraction forces are determined as a superposition of diffraction forces caused by the harmonic components of the irregular wave. The irregular wave is assumed to be a superposition of harmonic waves. It is assumed that the ship diffracting the waves is in its mean position. This is possible under the assumption that the diffraction phenomenon is described by a linear hydrodynamic problem. Such an approach significantly simplifies calculations because bulky computations can be performed at the beginning of the simulations and the ready solutions can be applied to determine the diffraction forces during the simulation.

The radiation forces are determined by added masses for infinite frequency and by the so-called memory functions given in the form of convolution. The memory functions take into account the disturbance of water, caused by the preceding ship motions, affecting the motion of the ship in the time instant considered (Cumminis, 1962).

The ways of solving 3D hydrodynamic problems and determining forces appearing in the equation of motion are presented in (Jankowski, 2006). The non-linear equations of motion are solved numerically.

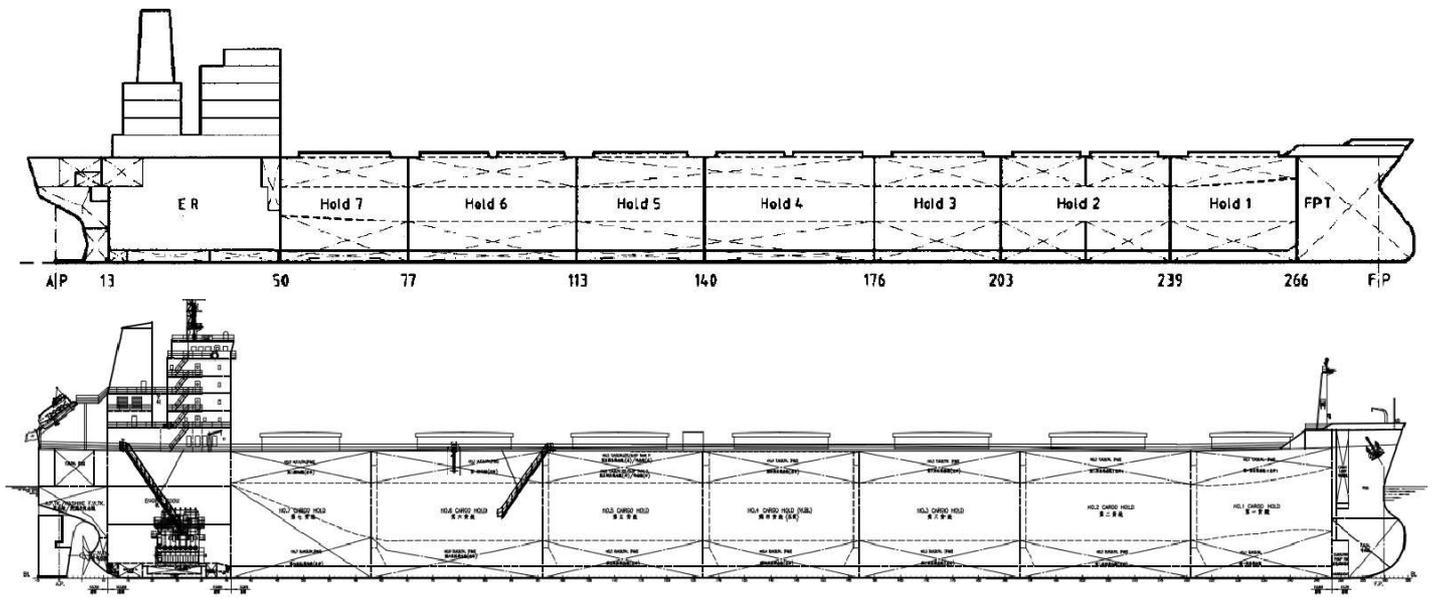


Figure 1 Two panamax bulk carriers of different design covered by the analysis; first built in 1977 and next in 2010.

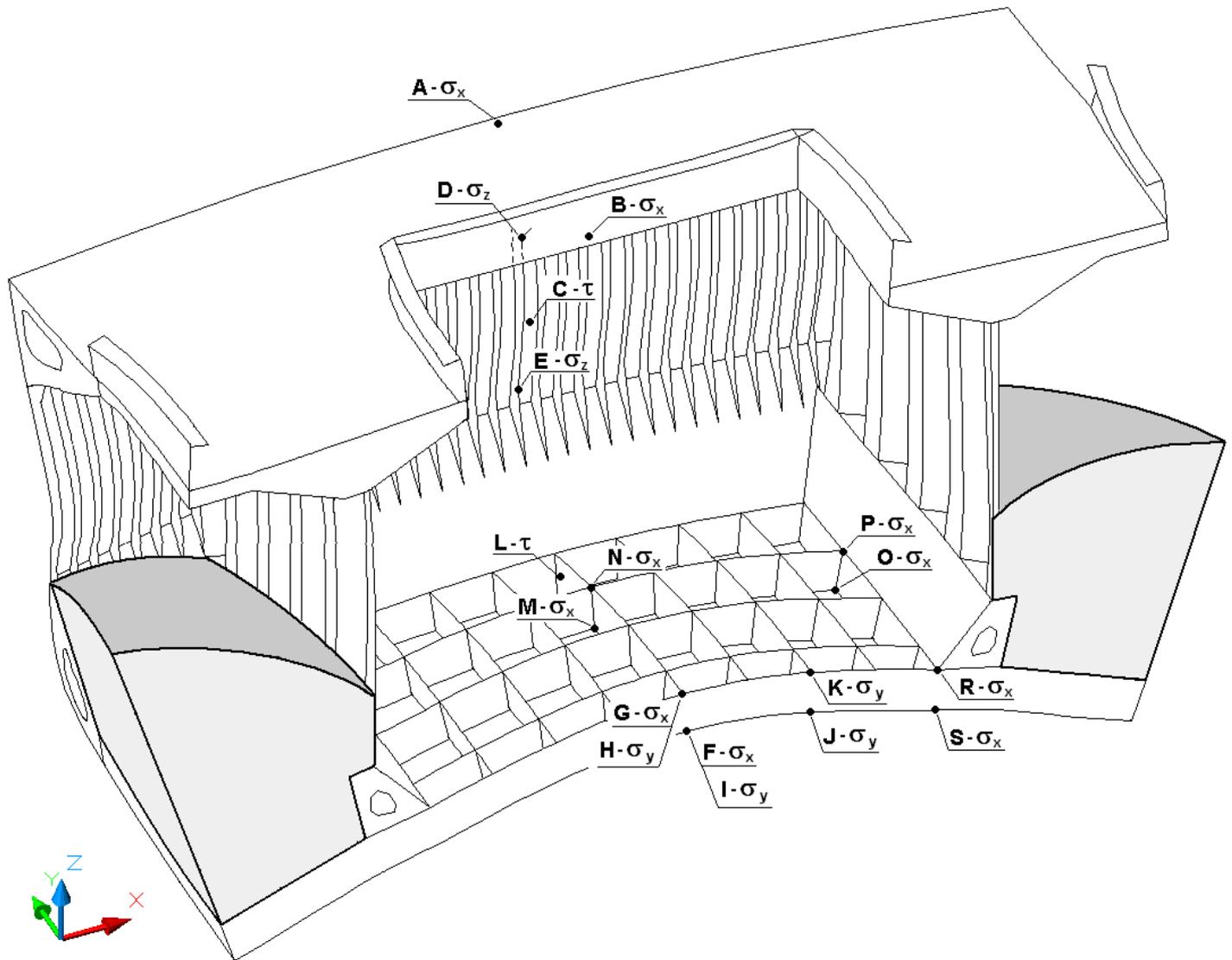


Figure 2 Members of the bulk carrier structure for which stresses were computed.

4 SIMULATION OF SHIP STRUCTURE RESPONSE TO IRREGULAR WAVES

The stress values in members of ship structure depend on the loads acting on the structure and the geometrical and material features of the structure.

The structure of two panamax bulk carriers of different designs has been analyzed. The ships are presented in Fig. 1; first a 1977 built ship, and next – a 2010 built ship.

The module of the middle part of each ship hull, embracing three holds, has been modeled to carry out the strength analysis using finite element method (FEM) – Fig. 3. The stresses have been computed for 58 structure members on both ship sides, but only the members for which the computed stresses were significant have been marked in Fig. 2. Stresses have been computed for alternate loading of the ships: the middle hold was empty and the neighboring ones were loaded with heavy cargo (Fig.2). This cargo rests on inner bottom, bilge tanks and lower stools of the bulkheads

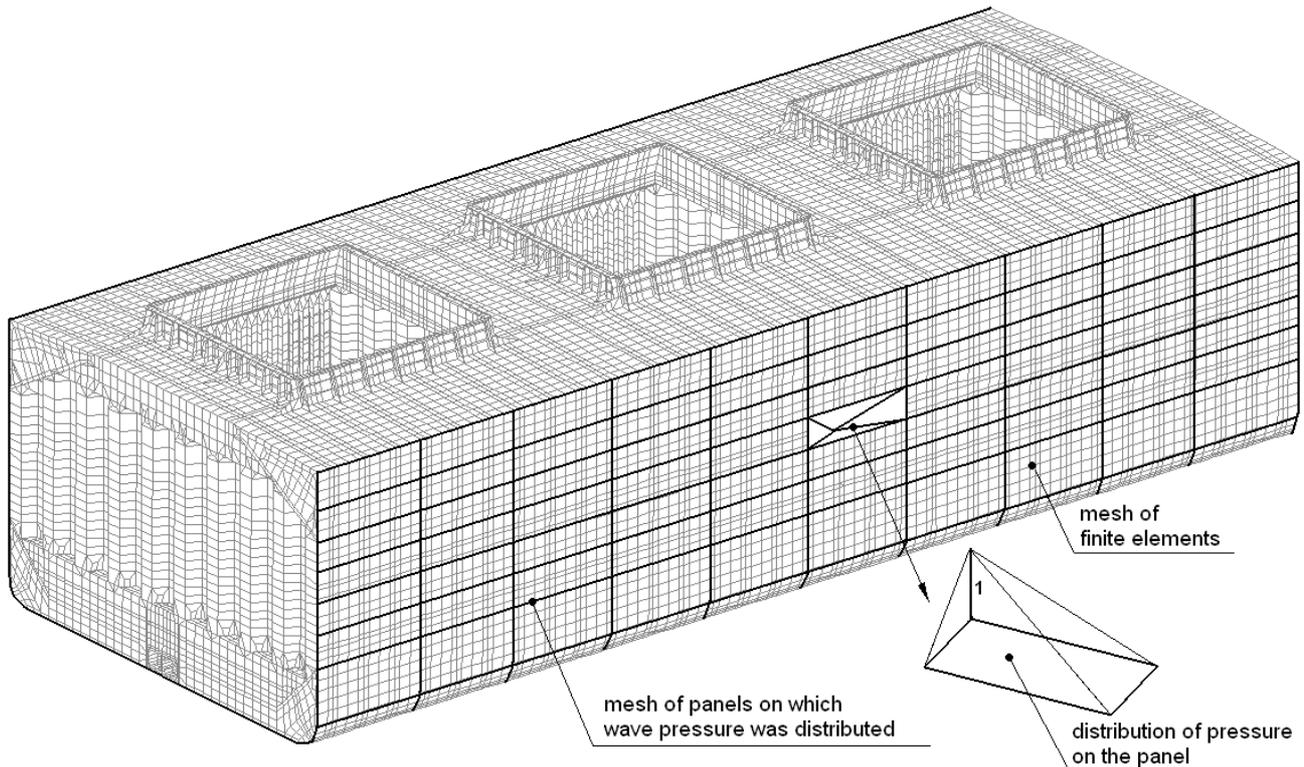


Figure 3 Ship hull module used to compute the influence of coefficients' values, and then to compute the stresses in chosen structure members.

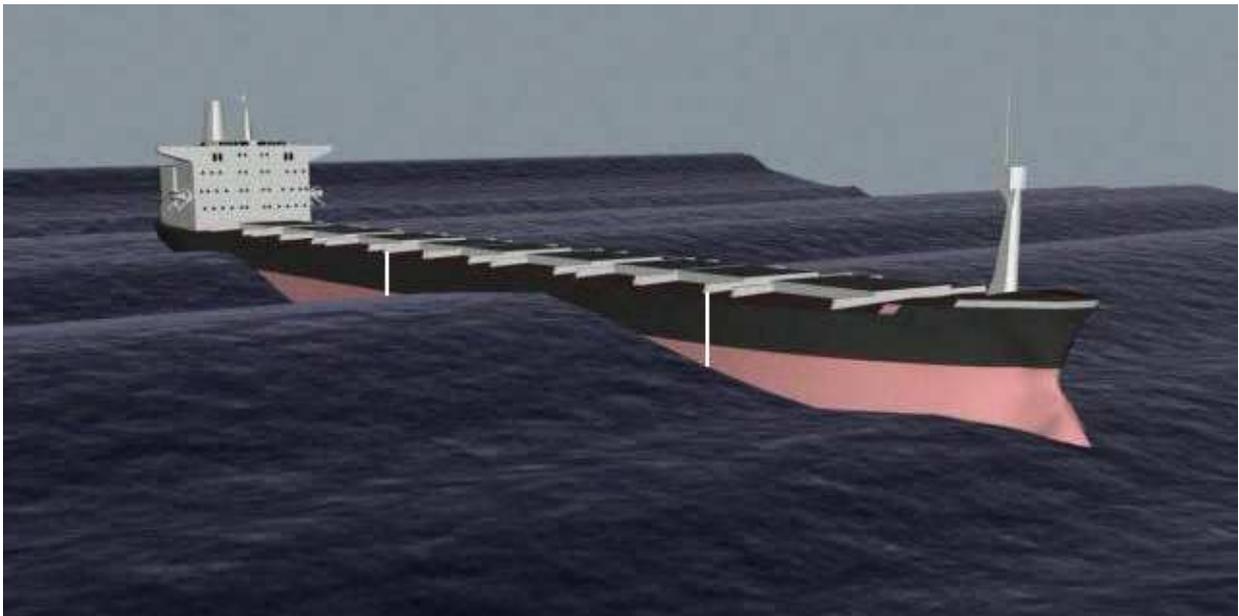


Figure 4 Wave generated loads on wetted surface of bulk carrier built in 1977; the position of hull module presented in Fig. 3 is marked.

The impact of the fore and aft parts of the hull, which are not part of the module, is accounted for in the form of the wave vertical and horizontal bending moments, generated at the ends of the cross section $x=x_{aft}$ and $x=x_{fore}$ of the considered hull structure module.

So, the loads of the of the structure of ship module consist of:

- vertical and horizontal bending moments applied at the ends of the ship module;
- pressure generated by water flow on the instantaneous wetted surface of the moving ship (Fig.4); and
- pressure generated on the inner bottom, bilge tanks and the lower stools of the bulkheads caused by gravitation and inertial forces acting on the cargo.

The simulation (PC computing) of the stresses in the structure members using the hull module (Fig. 3) consisting of approximately 100 000 finite elements is practically impossible as it requires:

- fifteen minutes of computations of stresses using FEM for above listed loads in one time step of the simulation;
- the simulations of the stresses in an irregular wave (sea state); it requires computations for thousands of time steps and;
- the long-term computations; it requires the simulations of the stresses for hundreds of sea states.

Therefore, the following method has been adopted to make the simulations of stresses in the structure of two bulk carriers possible:

The stress value at a selected structure member is calculated according to the formula (Jankowski, Bogdaniuk, 2007):

$$\sigma = \sum_i p_{1i}W_{1i} + \sum_k a_{2k}W_{2k} + \sum_l M_l W_{3l} \quad (3)$$

where W_{1i} = the influence coefficients of the wave excited pressure values on wetted surface of the ship; W_{2j} = the influence coefficients of pressure values on inner bottom, bilge tanks and stools, generated a_{2k} ; W_{3k} = the influence coefficient values of vertical and horizontal bending moments, generated at the ends of the cross section of the considered hull structure module; p_{1i} = the wave excited pressures on the wetted ship surface (Fig. 4); $a_{2k} = g + a_{vk} =$ gravitational acceleration and acceleration generated by ship motion and acting on the cargo in loaded cargo holds (Fig. 2), transformed in the computations to the pressure $p_{vk} = \rho a_{2k}h$ acting on the inner bottom, stools and bilge tanks, where h is the height (distance) of the cargo surface from the inner bottom, stools and bilge tank plating; M_l = vertical and horizontal bending moments, $l=1,2$, generated at the ends of the cross section $x=x_{aft}$ or $x=x_{fore}$ of the considered hull structure module.

The influence coefficients values W_{1i} , W_{2k} , W_{3l} of one structure member for fix i , k , and l are calculated applying FEM to the ship hull module (built of shell, beam and rod finite elements) and applying:

- loads distributed on a smaller number of panels than the number of finite elements (see Fig. 3) to reduce the time of computation.
- the unit pressure p_{1i} , or p_{2i} at a panel corner and linearly distributed over the panel as shown in Fig. 3, and for unit bending moments M_l $l = 1, 2$ at the ends of the cross section of the hull structure module.

One set of coefficient W_{1i} , W_{2k} , W_{3l} for fix i , k , and l is determined for one panel "corner". Values of these coefficients are computed before performing the simulation of ship dynamics in waves.

The combination of unit pressure distributions for four corners of the panel multiplied by actual pressures results in linear distribution of actual pressure over the panel; further multiplied by influence coefficients values gives stress which is a contribution of such loaded panel to the total stress in the selected structure member. Summing the contributions of stresses, according to formulae (3), from all panels and bending moments gives the stress value in the structure member. This procedure is used in each time step of the simulation.

Computations of the stresses generated by the loaded panel are appropriately organized in the computer program to reduce the time of computations. During the simulation, the actual values of pressures, accelerations and moments: p_{1i} , a_{2k} , M_l , are changing with time. Applying the above presented procedure the stress values in the considered structure member is simulated.

For example, the time history of stresses in the structure member A of the first ship (Fig.1) are presented in Fig. 5.

An example of the time history of vertical $M_v(x,t)$ and horizontal $M_h(x,t)$ bending moments in the cross section $x=0$ of the first ship are presented in Fig. 6, and Fig. 7. The presented moments are wave generated bending moment.

The next example, the time history of stresses in the structure member E of the first ship (Fig. 1) are presented in Fig. 8 .

The time history of wave generated pressure p at the bottom in the ship symmetry plane and at the ship side; and wave vertical acceleration a_v acting on the cargo in the aft hold are shown in Fig. 9 and 10.

The time intervals in the figures, for which the simulations are presented, contain instant t_{max} of maximum/minimum value of σ_{max} . This value is the maximum/minimum value of all sea states (for all simulations).

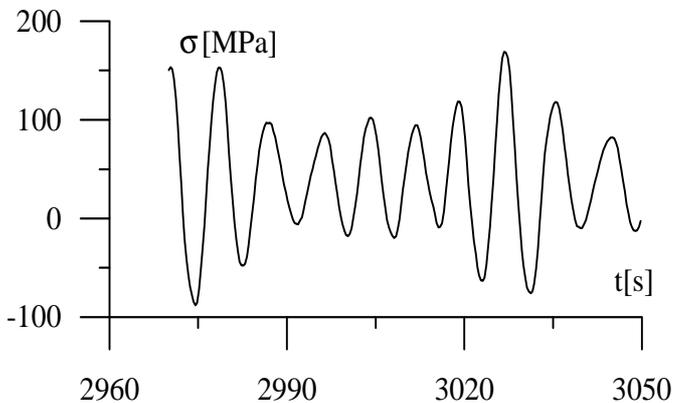


Figure 5 Time history of stresses σ_x in point A shown in Fig. 2, generated by irregular wave determined by: $H_s = 6.5$ m, $T_o = 7.5$ s and $\beta = 120^\circ$.

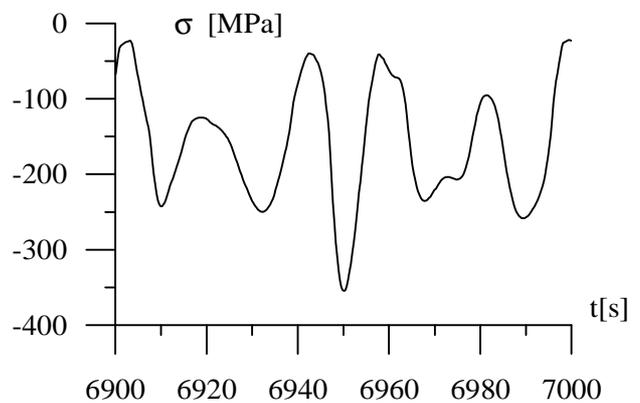


Figure 8 Time history of stresses σ_z in point E shown in Fig. 2, generated by irregular wave determined by: $H_s = 15.5$ m, $T_o = 15.5$ s and $\beta = 180^\circ$.

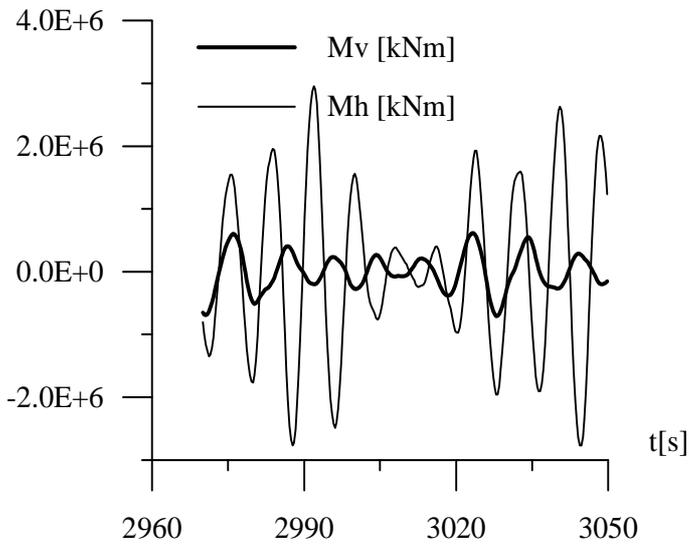


Figure 6 Time history of vertical M_v and horizontal M_h bending moments in the cross section at midship, generated by irregular wave determined by: $H_s = 6.5$ m, $T_o = 7.5$ s and $\beta = 120^\circ$.

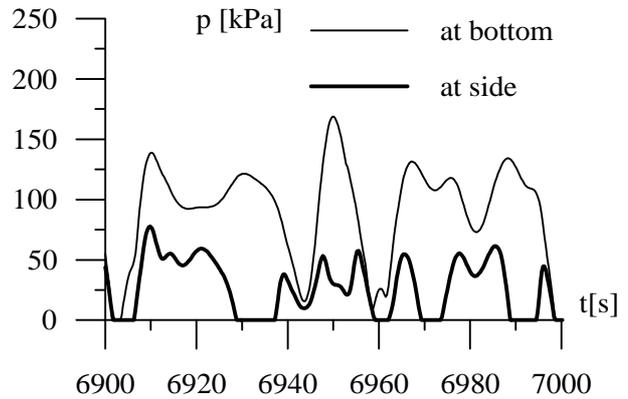


Figure 9 Time history of wave generated pressure p at the bottom and at the side (bold line), generated by irregular wave determined by: $H_s = 15.5$ m, $T_o = 15.5$ s and $\beta = 180^\circ$.

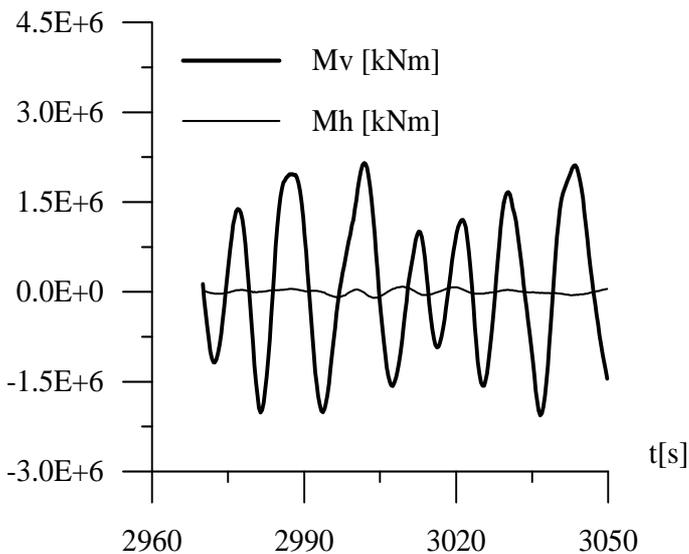


Figure 7 Time history of vertical M_v and horizontal M_h bending moments in the cross section at midship, generated by irregular wave determined by: $H_s = 15.5$ m, $T_o = 11.5$ s and $\beta = 180^\circ$.

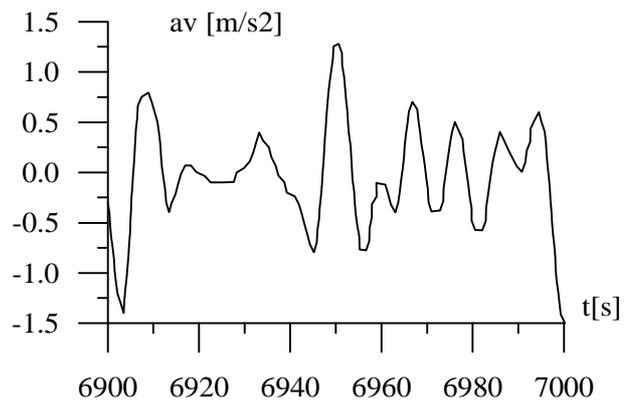


Figure 10 Time history of acceleration a_v in loaded aft hold, generated by irregular wave determined by: $H_s = 15.5$ m, $T_o = 15.5$ s and $\beta = 180^\circ$.

5 PROBABILITY DISTRIBUTIONS AND THEIR PARAMETERS

The numerical long-term probability density functions of stress σ (as the random variable) in the considered structure member were computed according to formula (2) taking into account the following assumptions:

$p_m = 1$, as only the alternate loading condition of the ship was considered;

$p_l = 1$, as according to the IMO Goal-Based Standards the areas A_l , $l=1$, should cover only the North Atlantic area;

$p_k =$ uniform probability distribution in the interval $[0, 2\pi]$, representing the ship course in relation to waves in the sea area A_l was used in the computation of the long-term probability density functions;

$p_{ij} =$ the probability of sea state occurrence in the North Atlantic; IACS Rec. 34 was used in the computations of the long-term probability density functions;

$f_{ijk} =$ the short term probability density function of random variable σ is approximated by a step function obtained by dividing the number of extremes (separately one minimum and one maximum in a stress cycle) belonging to the set interval of stress $\Delta\sigma$ by the total number of extremes occurring in a sea state (represented by one its realizations – an irregular wave) and by the length of the interval $\Delta\sigma$.

$$f_{ij}(\Delta\sigma) = \frac{n}{N|\Delta\sigma|} \quad (4)$$

where $n =$ number of maximums or minimums in the stress interval $\Delta\sigma$; $N =$ total number of maximums or minimums in the time of simulations of one realization of stresses in a chosen structure member in a given sea state; one stress maximum or minimum is per one stress cycle.

The examples of numerical long-term probability density function, computed according to (2), are presented in Fig. 11, Fig. 12 and Fig. 13.

The probability density function of stress σ in the considered structure member enables the computation of its characteristic values, for example, the stress value for the given probability (e.g. 10^{-8}) of exceeding this stress value. The process of computation also enables the selection of:

- the sea state defined by (H_s, T_o) and
- the angle between ship course and wave propagation (Fig. 14);

from all sea states and ship courses accounted for in the simulations that demonstrate the extreme value of stress σ in the considered structure member. They are called the “extreme load cases”. All together 640

cases (sea states and ship courses) were used in the computation process.

The results of computations of the characteristic values of long-term probability density function of stress σ in a selected structure members for two bulk carriers of panamax size of different designs are presented in Table 1 and 2. The “extreme load cases” are also presented in these tables.

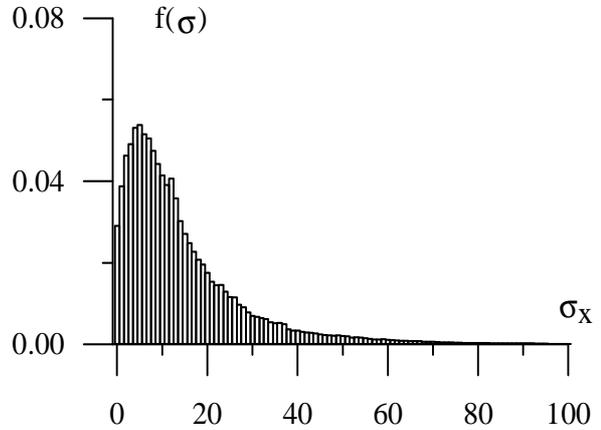


Figure 11 Probability density function of stresses σ_x in structure member A shown in Fig. 2.

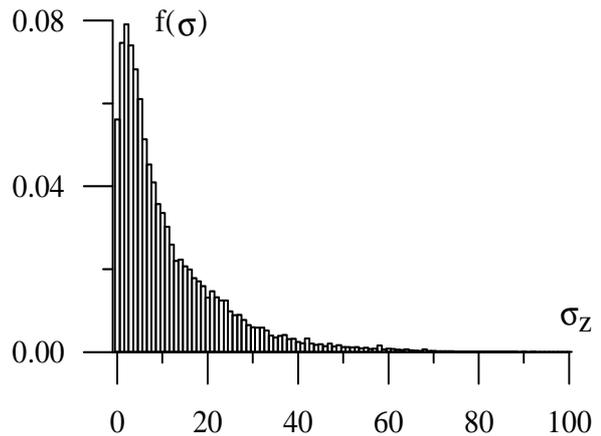


Figure 12 Probability density function of stresses σ_z in structure member E shown in Fig. 2.

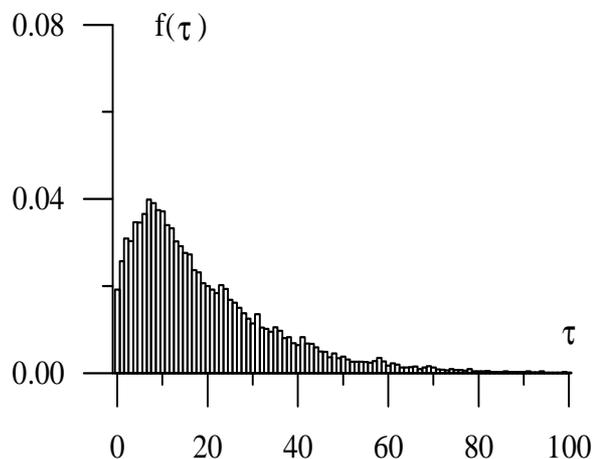


Figure 13 Probability density function of stresses τ in structure member C shown in Fig. 2.

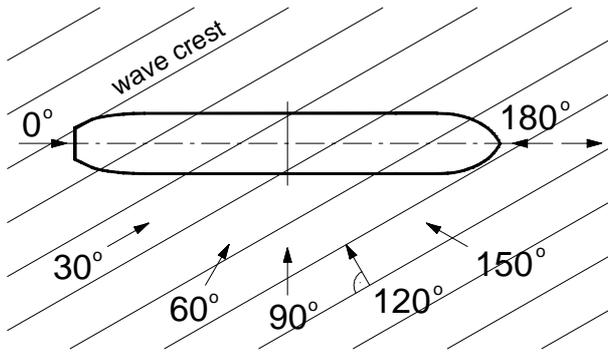


Figure 14 Definition of the angle between the ship course and wave propagation.

Table 1 The stresses in selected structure members and load cases for first panamax bulk carrier.

Struct memb.	σ_{pred} [MPa]	σ_{still} [MPa]	T_a [s]	β [o]	H_s [s]	T_o [s]	t_{max} [s]
A	167	41.27	9.74	120	6.5	7.5	3033
B	148	22.73	10.07	180	15.5	11.5	9925
C	-276	-2.37	10.63	180	16.5	12.5	8946
D	-57	-3.3	10.72	180	15.5	11.5	6772
E	-180	-139.68	11.48	180	15.5	15.5	6954
F	-111	-110.56	10.94	180	16.5	12.5	2791
G	95	103.12	10.51	180	16.5	12.5	4773
H	31	50.24	11.84	180	15.5	15.5	2877
I	-68	-73.74	11.91	180	15.5	15.5	9025
J	-72	-62.44	11.91	180	15.5	15.5	9025
K	13	27.4	10.7	180	15.5	15.5	9035
L	-63	-75.09	11.8	180	15.5	15.5	7990
M	-105	-79.33	10.03	180	16.5	12.5	1853
N	119	49.02	9.68	120	6.5	7.5	6804
O	-123	-25.93	9.83	120	6.5	8.5	2217
P	-128	-3.82	9.79	120	6.5	8.5	4260
R	71	5.66	10.47	180	16.5	12.5	1810
S	-83	-25.55	10.65	180	16.5	12.5	2791

Table 2 The stresses in selected structure members and load cases for second panamax bulk carrier.

Struct. memb.	σ_{pred} [MPa]	σ_{still} [MPa]	T_a [s]	β [o]	H_s [s]	T_o [s]	t_{max} [s]
A	233	129.43	9.89	120	6.5	8.5	3332
B	143	110.93	10	180	14.5	9.5	8385
C	193	3.46	10.64	180	14.5	9.5	3265
D	-73	-20.7	10.12	120	6.5	8.5	1073
E	-65	-78.51	10.43	120	6.5	9.5	5991
F	-65	-109.72	11.71	180	15.5	13.5	1477
G	12	28.76	11.15	180	15.5	13.5	2425
H	81	21.64	10.33	180	15.5	11.5	5433
I	-112	-170.06	10.82	180	14.5	9.5	3257
J	-85	-114.87	10.85	180	15.5	11.5	6459
K	-68	-25.2	10.54	180	16.5	12.5	4938
L	41	57.73	11.79	180	15.5	11.5	9062
M	-40	-81.85	11.4	180	15.5	13.5	7635
N	-19	-8.18	11.63	180	14.5	9.5	7346
O	-147	-84.72	10.2	60	6.5	8.5	2536
P	-151	-70.95	10.06	60	6.5	8.5	2536
R	-76	-86.28	10.94	180	16.5	12.5	5265
S	-63	-58.88	10.59	180	15.5	11.5	3353

The following denotations are used in the tables: σ_{pred} = the stress which can be exceeded with the probability equal to 10^{-8} ; σ_{still} = the stress in still water; T_a = the mean period of stress oscillations; β = the angle between the ship course and wave propagation (Fig. 14); H_s = the significant wave height; T_o = the average zero up-crossing wave period; and t_{max} = the instant of time in which the maximum/minimum stress value $\sigma_{max/min}$ occurs in the sea state defined by (H_s, T_o, β)

The $\sigma_{max/min}$ value is the maximum/minimum value for all studied sea states (all simulations).

6 DISCUSSION OF THE RESULTS AND CONCLUSIONS

The paper presents a method of long-term prediction of stresses in ship structure. This method is based on:

- scatter diagram representing the probability of sea states occurrence in the North Atlantic;
- simulation of ship motion in waves and its structure response (stresses) to the waves;
- assumption that the probability distribution representing the ship course in relation to waves propagation is the uniform probability distribution in the interval $[0, 2\pi]$;
- assumption that the master of the ship exercises good seamanship.

In the present class rules only the sea area of the North Atlantic – the area featuring the most severe wave conditions, is recommended to use to provide the safety margin.

150 sea states determined by the scatter diagram (IACS Rec. 34) and seven ship courses in relation to waves ($\beta = 0^\circ, 30^\circ, 60^\circ, \dots, 180^\circ$) are taken into account in the long-term procedures of determining stresses in ship structure (symmetry is taken into account). All together 640 simulations for each of the two panamax type bulk carriers (Fig. 1) have been carried out. This number is below 150 multiplied by 7 in the result of the assumed impact of the shipmaster's decision to change ship course in relation to the wave propagation in heavy sea states. For example, it was assumed that in the result of master decision for $\beta = 120^\circ$ the significant wave never exceeds 6.5m ($H_s \leq 6.5m$) and probability of ship meeting in such waving conditions was modified respectively. Similar modifications were made for other β values, showing that the limiting of H_s depends on β . No limitation on H_s was made for $\beta = 180^\circ$.

However, such an approach cannot be used in design practice. Therefore, a selection of wave load cases, representing the long-term structure response to waves for certain class of ships (e.g. panamax

bulk carriers), should be made. The computations presented in Table 1 and 2 shows that:

- In the first ship (Fig. 1), built in year 1977, the highest stress values occur in the side of the ship: in frame flange (structure member E – Fig. 2) $\sigma_z = \sigma_{\text{pred}} + \sigma_{\text{still}} = -320$ MPa, and in side plate (structure member C – Fig. 2) $\tau = -278$ MPa. These values exceed the yield stress equal to 235 MPa and allowable shear stress equal to 135 MPa of the steel used. That ship sank in the year 2000, in the North Atlantic. The sequence of events causing sinking of the ship started from breach of the side.
- In the second ship (Fig.1), built in year 2010, the highest stress value occurs in the deck plate at side: $\sigma_x = \sigma_{\text{pred}} + \sigma_{\text{still}} = -362$ MPa. This value exceeds the yield stress equal to 315 MPa of the tensile strength steel applied.

The case of stress in the deck plate at shipside (A– Fig. 2) shows that the highest stresses are generated in the sea state ($\beta = 120^\circ$, $H_s = 6.5$ m, $T_o = 8.5$ s). This is the result of contribution of the horizontal bending moment M_h to the stress, much bigger than the contribution of the vertical wave and still water bending moments: $M_v + M_s$. In sea state ($\beta = 180^\circ$, $H_s = 15.5$ m, $T_o = 11.5$ s) horizontal bending moment M_h is negligible (see Fig. 6 and 7).

Analyses of the results of the computed strength ranges, presented above, shows that for panamax bulk carriers it is sufficient to take into account only two wave load cases: ($\beta = 120^\circ$, $H_s = 6.5$ m, $T_o = 8.5$ s) and ($\beta = 180^\circ$, $H_s = 15.5$ m, $T_o = 11.5$ s).

The sea state ($H_s = 15.5$ m, $T_o = 12$ s) is one of the highest in the scatter diagram applied. If the equivalent regular waves of periods $T_o = 12$ s and $T_o = 8.5$ s are assumed then the length of the wave is equal to $\lambda = 1.56T_o^2/|\cos(\beta)|$, what in the first and the second wave case is approximately equal to the length of panamax bulk carriers (225m).

However, the equivalent regular wave should only be used, assuming that λ is equal to the length of the ship to determine the parameters of the irregular wave (wave load case). These parameters are used to determine parameters of the irregular waves (T_o , H_s) and next to simulate of ship response to waves (stresses) and to assess the strength of midship module with the use of the Finite Element Method.

The simulation of ship structure response to irregular wave takes into account the shift phase between different loads (pressures, accelerations, bending moments) in a strict manner and do not require the application of combination factors, presented in binding rules of classification societies.

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