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# Numerical solutions of nonlinear hydrodynamics problems with free surface

M.WARMOWSKA

Polski Rejestr Statków S.A., Al. Gen. Jozefa Hallera 126, 80-416 Gdańsk

The paper presents numerical solutions of hydrodynamics problems with free surface developed in Polski Rejestr Statków during last several years. Modelling phenomenon of ship motion in waves, sloshing inside tanks or flooded holds and flow of trapped water on vessel deck requires special assumptions and algorithms. Accuracy of these methods significantly affects the solutions. The paper contains comparison of various methods. The studies contribute to better awareness of ship behaviour, capsizing phenomenon, failure of ship structure and other undesired events.

Keywords: Free surface problem, Harmonic wave, Sloshing, Shallow Water Problem, BEM

## 1. Introduction

Volume of Fluid technique[1], Shallow water method [2], Boundary Element Methods[3], Finite Elements Method [4], and others [5] are used to solve the hydrodynamics problems. It is important to choose a model as simple as possible but without loosing sight of the essence of water behaviour. The proper determination of hydrodynamic forces acting on the moving ship in waves is very important in terms of safety.

Most hydrodynamics problems are solved with linear or quasi-linear assumptions. In determining the forces acting on the objects submerged in water the elevation of waves is neglected or the elevation is obtained from the linear part of Bernoulli equation.

Such phenomena as:

- water flow inside ship’s tank or damaged hold,
- water trapped on vessel’s deck,
- water flow around small objects

need more sophisticated methods, which describe the free surface elevation and consequently pressure exerted on the body with more accuracy.

## 2. Standing wave – the shift of the free surface

The solution of the 2D water flow problem inside ship tank focuses on determining the free surface. The shape of the liquid with constant mass keeps changing. First of all the pressure and velocity field depend on the position of disturbed surface of the fluid. The comparison of linear and non linear methods was made [6]. Non linear numerical methods have to be used to solve the problem of fluid motion inside tanks (the free surface reached the ceiling).

The strict solution of the linear problem of standing wave inside a rectangular tank is known. The potential $\phi_S$ of velocity flow is given by the equation:

$$\phi_S(x, y, t) = \frac{g r_0}{\omega} \frac{\cosh(k(y+H))}{\cosh(kH)} \cos(kx)\cos(\omega t) \tag{1}$$

For a small amplitude $r_0$ the components ($u_s$, $v_s$) of velocity take the form:

$$\nabla\phi_S = (u_S, v_S) = \left( -\frac{gkr_0}{\omega}\frac{\cosh(k(y+H))}{\cosh(kH)}\sin(kx)\cos(\omega t), \quad \frac{gkr_0}{\omega}\frac{\sinh(k(y+H))}{\cosh(kH)}\cos(kx)\cos(\omega t) \right) \tag{2}$$

where
$H$ – filling depth,
$k$ – wave number,
$\omega$ – frequency of the standing wave oscillation,
$g$ – acceleration of gravitation.
The free surface for linear solution can be obtained from the equation:

$$\xi_S = -\frac{1}{g}\frac{\partial\phi_S}{\partial t} \tag{3}$$

and has the following form:

$$\xi_S(x,t) = r_0\cos(kx)\sin(\omega t), \qquad \mathrm{y(t) = 0}\,. \tag{4}$$

This solution of the linear problem was compared with results obtained using three methods for free surface shifting, Fig 1. It was assumed that for start time $t$ equal zero the velocity field is given by formulae (2). Initially free surface is undisturbed and the position of the free surface and potential are determined numerically using various methods Fig.2.

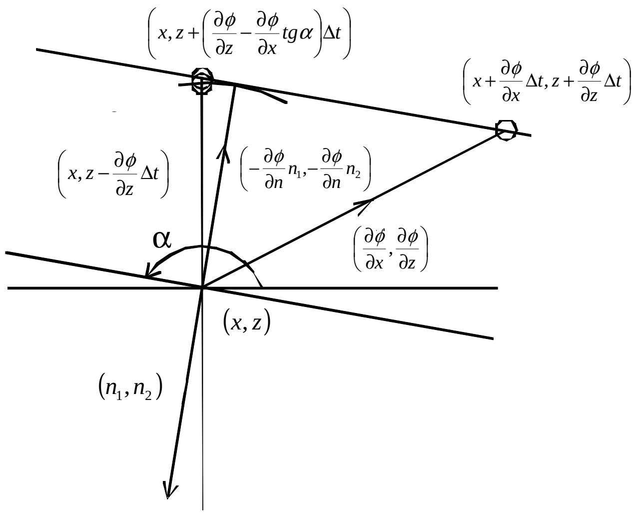


Fig. 1 Methods of a free surface shift

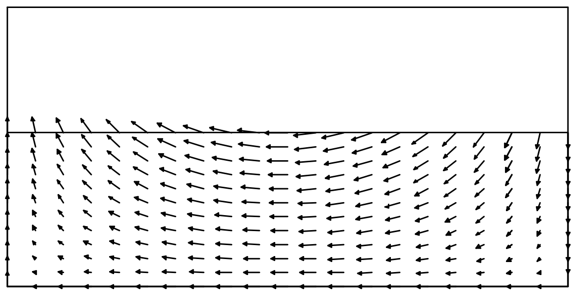

Fig. 2 Initial conditions

**Method I**

In this method the points describing free surface are shifted for fixed $x_i$, $i$=1,..,N, where $x_i$ is the first coordinate of the free surface point.

The equations determining the free surface elevation take on the forms:

$$x_i(t) = x_i$$
$$\frac{\partial \xi}{\partial t}(x_i(t),t) = -\frac{\partial \phi}{\partial n}(x_i, \xi(x_i,t),t)\sqrt{1+\left(\frac{\partial \xi(x_i,t)}{\partial x}\right)^2} \qquad t \in (t^n, t^{n+1}] \tag{5}$$

**Method II**

This method was obtained from Method I. If value $\frac{\partial \xi(x_i,t)}{\partial x} \approx 0$ then the term $\sqrt{1+\left(\frac{\partial \xi}{\partial x}\right)^2}$ can be neglected. The equations determining the free surface elevation have the form:

$$x_i(t) = x_i \qquad t \in (t^n, t^{n+1}]$$
$$\frac{\partial \xi}{\partial t}(x_i(t),t) = -\frac{\partial \phi}{\partial n}(x_i, \xi(x_i,t),t). \tag{6}$$

**Method III**

In this method the free surface is moved according to formula:

$$\frac{dx}{dt} = \frac{\partial \phi}{\partial x}, \quad \frac{dz}{dt} = \frac{\partial \phi}{\partial z} \tag{7}$$

Each of the methods mentioned above imply a different formula determining the derivatives of velocity potential $\phi$ with respect to time *t*. In the last method the velocity potential is obtained from the differential equation defined as follows:

$$\frac{\partial \phi}{\partial t} = \frac{1}{2}\left|\nabla \phi\right|^2 - gz \tag{8}$$

Accuracy of methods I and II depends on the accuracy of numerical determination of the derivation $\frac{\partial \phi}{\partial n}$ (n – normal vector directed into the fluid).

Fig. 3 presents the free surface position obtained after time *t* equals *T*/4 (*T* – a period of motion). Method III gives the shape much similar to a shape obtained during a experiment.

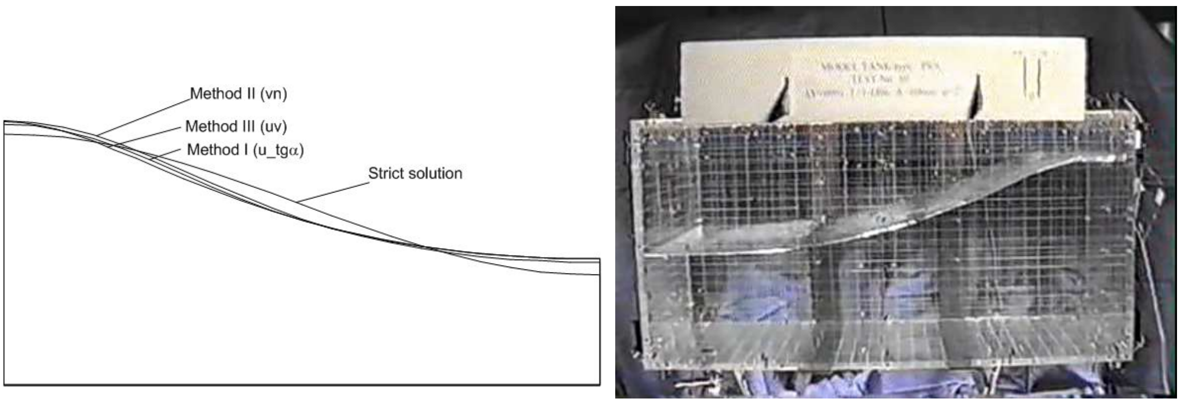


Fig. 3 The free surface shapes obtain by the methods and during an experiment

Numerical methods presented above are used in problems with free surface determination such as sloshing inside ship tank filled over 40%.

## 3. Regular wave – pressure

In the linear model the velocity potential $\phi$ of harmonic wave is defined as follows:

$$\phi = \frac{r_0}{\omega} e^{kz} \sin(kx - \omega t) \tag{9}$$

where $r_0$ is the amplitude of harmonic wave, $k$ is the wave number, ω is the frequency of harmonic wave oscillations ($\omega^2 = kg$ for a deep water), ($x$, $z$) is the position of water particle, $t$ is the time treated as a parameter. Axis OZ has an upward direction. Assuming small amplitude [6] of wave the pressure $p_I$ (for $z<=0$) can be obtained from the linear part of Bernoulli equation:

$$p_I = p_0 - \rho g z + \rho g \zeta e^{kz} \tag{10}$$

where $p_0$ is the pressure on the free surface, $\rho$ is the density of fluid, ζ is the elevation of the wave surface above a point on undisturbed surface ($x$,0) defined as follows:

$$\zeta = r_0 \cos(kx - \omega t). \tag{11}$$

Faltinsen [7] proposed to calculate pressure $p_{II}$ with respect of wave trough or wave crest (Smith's effect). The pressure is calculated from the still water level as the point of reference. For a wave crest the pressure is extended, for a wave trough the hydrodynamic pressure is disregarded. For deep water condition the pressure is given by formula:

$$p_{II} = p_0 - \rho g z + \rho g \zeta e^{k(z-\zeta)}. \tag{12}$$

Quite different results are obtained when the motion around average position ($x_0$, $z_0$) of a water's particle is assumed, Fig. 4.

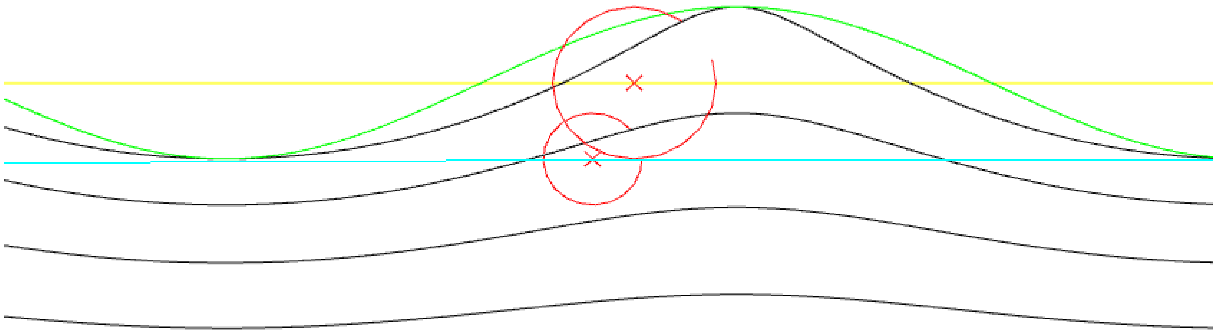

Fig. 4 Orbital motion – position of surfaces for liquid particles with fixed $z_0$ and snapshot time $t$

In Fig. 4, black lines show the surfaces created by cycling liquid particles for fixed $x_0$,$z_0$ (a yellow line is the still water level for particles with the vertical average position $z_0$ equal to zero and a blue line for $z_0$ equal to $-r_0$), the green line is the free surface determined by equation (11), red cycles are tracks of liquid particles determined by equations(13).

In the third method (presented in report [8]) the wave is described as a cycling particles around their average position:

$$x(t) = x_0 - r_0 e^{kz_0} \sin(kx_0 - \omega t), \tag{13}$$

$$z(t) = z_0 + r_0 e^{kz_0} \cos(kx_0 - \omega t).$$

In deep water it is the orbital motion. In this method the position of free surface is defined by equation (13) for $z_0$ equal to zero. If the average position ($x_0$, $z_0$) of a liquid particle is known then the value of pressure $p_{III}$ is obtained from the following formulae:

$$p_{III} = p_0 - \rho g(z_0 + \zeta_0) + \rho g \zeta_0 e^{kz_0} - \rho g 0.5kr_0^2\left(1 - e^{2kz_0}\right) \tag{14}$$

where the elevation of the wave surface $\zeta_0$ is defined as:

$$\zeta_0 = r_0 \cos(kx_0 - \omega t). \tag{15}$$

In the third method the average position ($x_0$,$z_0$) of particles must be know to determine the free surface position and pressure. This method gives pressure not only for z<=0 but also for particles located close to the free surface. For example the value of pressure for z>0 is important in case of water trapped over bulwark.

If the wave amplitude $r_0$ is small the pressure $p_{IV}$ may be approximated by the following formulae:

$$p_{IV} \approx p_0 - \rho g(z_0 + \zeta_0) + \rho g \zeta_0 e^{kz_0} \tag{16}$$

Table 1 shows the pressures obtained using these four methods for the following wave parameters:

- the amplitude $r_0$ is equal to 2,0 meters,
- the period $T$ of wave oscillation equals 10,12 seconds,
- the average position ($x_0$, $z_0$) is situated at a point (0m,–2,5m).

The position of oscillating liquid particle ($x$, $z$) is calculated using equation (13). The wave elevation $\zeta$ in method I and II is determined by equation (11) and the wave elevation $\zeta_0$ in method III and method IV – by equation (15).

The relative error between method I and method III reaches 14,56% (for $t$ equal to zero). The differences between method II and method III is not greater than 9%. The relative errors will increase if the cycling particle is reaching the wave surface and the errors will decrease if the cycling particle moves away from the wave surface. The fourth method, which is a simplified version of the third method, provides sufficient approximations with the relative error not exceeding 1%.

The pressure calculations are simpler in the first and second method than in the third method. Methods I and II are usually used to solve problems when the main dimensions of the vessel exceed significantly the wave amplitude. In the case of fishing vessel motion, when the vessel draught is about 1 to 3 meters, the third method (or fourth method), describing the orbital motion of the water particles, should be used [9].

**Table 1**: Pressure and relative errors obtained by the four methods

| t [s] | x[m] | z[m] | $\zeta$[m] | $\zeta_0$[m] | $p_I$[Pa] | $p_{II}$[Pa] | $p_{III}$[Pa] | $p_{IV}$[Pa] | $\lvert p_{III}-p_I \rvert / p_{III}$ [%] | $\lvert p_{III}-p_{II} \rvert / p_{III}$ [%] | $\lvert p_{III}-p_{IV} \rvert / p_{III}$ [%] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.00 | -0.69 | 2.00 | 2.00 | 26.48 | 25.00 | 23.12 | 23.26 | 14.56% | 8.17% | 0.61% |
| 0.84 | 0.91 | -0.93 | 1.77 | 1.73 | 26.48 | 25.33 | 23.37 | 23.51 | 13.30% | 8.39% | 0.60% |
| 1.69 | 1.57 | -1.59 | 1.10 | 1.00 | 26.46 | 26.02 | 24.06 | 24.20 | 9.98% | 8.14% | 0.59% |
| 2.53 | 1.81 | -2.50 | 0.14 | 0.00 | 26.43 | 26.43 | 25.00 | 25.14 | 5.75% | 5.72% | 0.56% |
| 3.37 | 1.57 | -3.41 | -0.89 | -1.00 | 26.41 | 26.13 | 25.94 | 26.08 | 1.83% | 0.75% | 0.54% |
| 4.22 | 0.91 | -4.07 | -1.70 | -1.73 | 26.40 | 25.40 | 26.63 | 26.77 | -0.86% | -4.62% | 0.53% |
| 5.06 | 0.00 | -4.31 | -2.00 | -2.00 | 26.39 | 25.00 | 26.88 | 27.02 | -1.81% | -6.97% | 0.52% |
| 5.91 | -0.91 | -4.07 | -1.70 | -1.73 | 26.40 | 25.40 | 26.63 | 26.77 | -0.86% | -4.62% | 0.53% |
| 6.75 | -1.57 | -3.41 | -0.89 | -1.00 | 26.41 | 26.13 | 25.94 | 26.08 | 1.83% | 0.75% | 0.54% |
| 7.59 | -1.81 | -2.50 | 0.14 | 0.00 | 26.43 | 26.43 | 25.00 | 25.14 | 5.75% | 5.72% | 0.56% |
| 8.44 | -1.57 | -1.59 | 1.10 | 1.00 | 26.46 | 26.02 | 24.06 | 24.20 | 9.98% | 8.14% | 0.59% |
| 9.28 | -0.91 | -0.93 | 1.77 | 1.73 | 26.48 | 25.33 | 23.37 | 23.51 | 13.30% | 8.39% | 0.60% |
| 10.12 | 0.00 | -0.69 | 2.00 | 2.00 | 26.48 | 25.00 | 23.12 | 23.26 | 14.56% | 8.17% | 0.61% |

## 4. Shallow water method

The shallow water method is used when horizontal components ($u_1$,$u_2$) of water velocity do not depend on the vertical coordinate $z$ and vertical component $u_3$ of water velocity is small (vertical acceleration is negligible). Such assumptions can be applied in modelling such phenomenon like the water motion inside low level filled ship tank or modelling of the water motion over the ship deck [9].

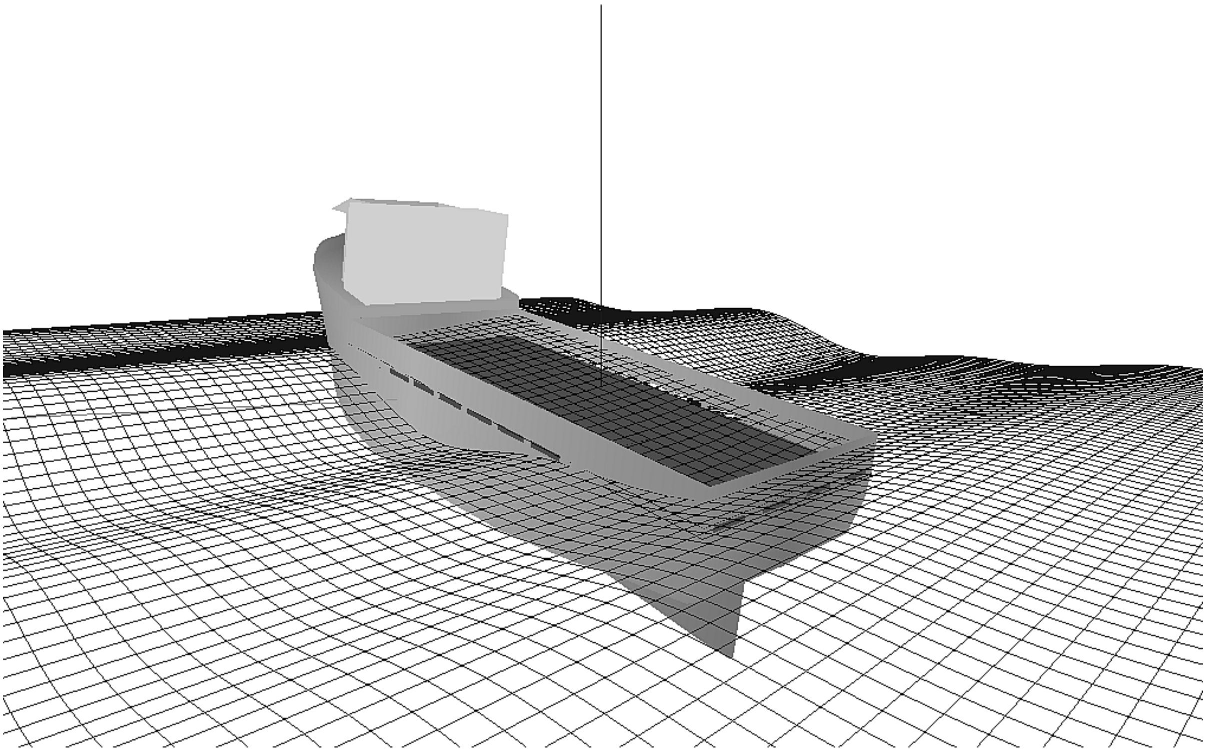

Fig. 5 Water motion over the deck

The algorithm of shallow water method comprises:

- free surface determination (water level $h$ over deck position $z_b$),
- calculation of pressure $p$, which is determined by the position of the free water surface and forces $f_3$ acting on the water:

$$p(x,y,z) = p_a + \rho \int_{z_b + h(x,y,z_b)}^{z} f_3(x,y,s)\,ds\,, \tag{17}$$

- calculation of the vertical velocity component $u_3$, obtained from Euler equation:

$$u_3(x,y,z) = \left( -\frac{\partial u_1}{\partial x}(x,y) - \frac{\partial u_2}{\partial y}(x,y) + q \right)(z - z_b)\,, \tag{18}$$

where $q$ represents change of water's mass on the hold bottom/deck.

The shallow water method is three dimensional and the solution depends on the position of free surface.

## 5. Conclusions

Methods presented in this paper prove that the correct determination of the shape of the free surface is important in modelling many hydrodynamic phenomena. Nonlinear solutions (pressure distribution and velocity) are significantly different from those obtained using linear simplifications.

The opportunity to determine the velocity field above the surface undisturbed, which is necessary for the proper designation of the forces acting on small vessels, is assured when the movement around the average position is adopted. The differences of pressure in the case of commonly used methods (pressure $p_I$ or $p_{II}$) and the method of movement around the average position (pressure $p_{III}$ lub $p_{IV}$) are quit big. The values of the forces acting on a floating object vary significantly depending on the method used.

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