Polski Rejestr Statków

HISTORY OF SHIP'S STATIC STABILITY CRITERIA DEVELOPMENT

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INTRODUCTION

The sea transport of people and goods is probably the biggest contributor to the world development. It started in ancient times and has been developed through the whole human history. Sea transport was affected by vessel's stability from the very beginning and the first problem which seafarers had to face was building the stable vessels.

Probably, at the beginning of shipping history, seafarers and shipbuilders used intuition when dealt with stability of vessels, basing on gathered experience. This allowed to built stable vessels and operate them safely. Fig. 1 presents now-a-day building of vessel without any technical plans on island Arwad, Syria, basing on tradition and intuition.

Fig. 1. Building of vessel on island Arwad, Syria

Obviously, it is impossible to apply such an approach to new types of ships and especially warships, to which new ideas and innovations are continuously introduced. The first who laid foundations for a quantitative assessment of vessel hydrostatic stability was Archimedes. He developed a stability measure similar to the righting arm.

The solution of the problem in terms of criteria for initial stability of vessels came 20 centuries later. In the mid of eighteen Bouguer working in France, and Euler working in Russia, independently came to equivalent solutions on ship stability theory. Bouguer developed the theory of ship metacentre which together with the position of vessel's center of gravity allowed to estimate stability. Euler stated that floating body inclined from equilibrium should have restoring moment to return it to the upright position and he derived mathematical form of this moment. Bouguer's and Euler's theories led to equivalent results.

The criteria developed by Bouguer and Euler referred to the initial stability. The metacentric height *GM* is used till now to evaluate the ship stability. However, it appeared that *GM* does not poses information on the "stability capacity of the ship", which is incorporated in the curves of stability (righting arm as a function of heeling angel). And the criterion on the "stability capacity of the ship" was not developed until Rahola published his doctoral thesis in 1939.

The discussed criteria refer to the stability of intact ship. Then on the agenda was put the stability criteria for damaged ships, which is another problem. There are still attempts to develop rational and more accurate damaged stability criterion to predict the capsizal resistance of damaged vessels.

Figures reported by International Maritime Organization(IMO) show that the annual loss of life on world's small ships, especially fishing vessels, are huge. There are many reasons of that situation, but the main one is capsizing of the vessel during moving in waves as the dynamic stability of vessels has not been standardized yet.

At the end of the short presentation of the history of stability criteria development, the modern mathematical theory of hydrostatics was applied to a simple body to illustrate the mechanism of vessel's stability.

ARCHIMEDES

First, who laid foundations for stability of ships was Archimedes (287 – 212 BC)[1]. He derived "stability measure, similar to the righting arm, and presented the theory for assessing the ability of floating inclined ship to right itself". In manuscript "On floating bodies" (Archimedes works, 2002) Archimedes made use of his earlier results referring to the equilibrium of moments, centers of quantities (areas, volumes, weights), center of gravity and a rule of shift of center of the system when some quantity is added. He also used quadrature – an inscribed polygonal, to approximate the area of truncated parabola.

Archimedes postulated the following properties of fluid at rest[2]:

> *"Let it be supposed that the fluid is of such character that:*
>
> *its parts lying evenly and being continuous,*
>
> *that part which is thrust the less is driven along by that which is thrust the more and*
>
> *that each of its part is thrust by the fluid which is above it in a perpendicular direction,*
>
> *unless the fluid is constrained by a vessel or anything else."*,

and derived the principle of hydrostatic equilibrium in the following form:

> *"Any solid lighter then fluid will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of fluid displaced."*

The proof of this law as presented in (Nowacki & Ferreiro, 2003) is very brief: "in equilibrium the solid is at rest and the fluid is at rest, thus if the body is removed from the fluid and cavity left its volume is filled with fluid matter, then the fluid can only remain at rest if the replacing fluid volume weights as much as the solid, else the fluid would not remain in equilibrium and hence at rest."

In modern fluid theory this Archimedes postulate and proposition is a conclusion from the Navier–Stokes equations, assuming that that the fluid and the floating body are at rest (see Appendix).

[1] All historic information from Archimedes to Euler and the references has been drawn from paper by Nowacki & Ferreiro, 2003

[2] All quotations are taken from Nowacki & Ferreiro, 2003

The vessel is stable if being inclined from the equilibrium position has ability to come back to equilibrium, which means that restoring moments must appear to restore the vessel to the equilibrium (see (A18) in Appendix).

To solve the problem of stability of hydrostatic equilibrium Archimedes used a segment of paraboloid of rotation (Fig. 2) of homogenous material whose specific gravity was less than that of fluid and evaluated geometrically the lever between the buoyancy and weight forces. He was able to find:

- the center *B* of underwater domain occupied by paraboloid inclined in such a way that its base was not immersed; and
- the center of mass *R* of the homogeneous solid.

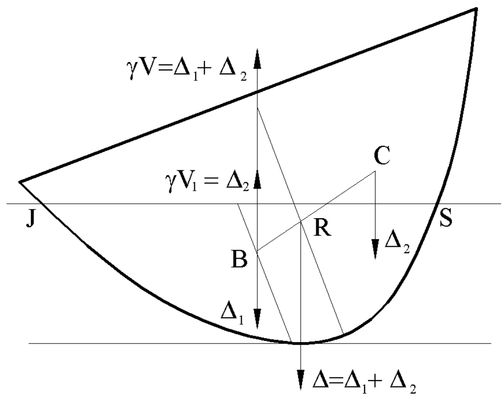

Fig. 2. Restoring moments, righting arms for inclined homogeneous paraboloid, based on Archimedes' "On floating bodies" (from Nowacki & Ferreiro, 2003)[3]

The buoyancy acts vertically upright through *B* (which Archimedes knew) and weight acts through *R* in the same direction but oppositely oriented. The projection of |*BR*| on the horizontal is the conventional righting arm. Instead of using this stability measure Archimedes, as described in (Nowacki & Ferreiro, 2003), cut this solid into abovewater Δ_2 and underwater Δ_1 parts, removed underwater part Δ_1 and corresponding equal share of the buoyancy force, which have no moment about *B*. Thus the weight Δ_2 acting through *C* and the opposite buoyancy force $\gamma v_2 = \Delta_2$, acting through *B*, constitute the couple force on the "righting arm". The center *C* was found from *B* and *R* by applying the center shift theorem when removing the underwater part from the system. This yields a positive "righting arm" for the force couple Δ_2 through *B* and *C*, restoring the solid to the upright position. Archimedes probably applied this method to better illustrate the restoring moment induced by the inclined solid.

Archimedes considered a very special case, however, he demonstrated physical principles of the hydrostatic stability problem for a finite angle

[3] All denotations are as on the figures in the referred papers

of inclination. But, what is important, he showed the idea, the model, which enable others, eighteen ages later, to start again the development of stability criteria.

OTHERS

Eighteen centuries after Archimedes, the Dutch scientist, Simon Stavin (1548 – 1620), was probably the first who introduced the concept of "hydrostatic pressure distribution" proportional to the weight of a water prism above the depth in question. This concept enabled him to calculate the water loads on ship sides. Today, pressure is one of the basic notions in hydromechanics (see Appendix).

The French mathematician, Poul Hoste (1652 – 1700) was the first who tried to quantify the problem of ship stability, what without calculus, which had not been yet formulated, was difficult and he made several errors. However, he described the inclining experiment of the ship to demonstrate the "force to carry the sails".

La Criox (1690 – 1747) correctly understood the role of weight and buoyancy forces acting in the same vertical line in ship equilibrium without external moment and followed Archimedes in his stability criterion derivation by examining the body heel and requiring positive righting arms, but he incorrectly perform the calculations.

The problem which affected the stability of ships were the gun-ports introduced in early 1500s. "This greatly increased the fire power of naval ships, but brought two problems; the ships got much heavier, and with large holes in the ship side the available freeboard got dramatically smaller"[4]. The tragic capsizing of two naval ships: Mary Rose (1545) and Vasa (1628), are good examples of unrecognized areas of stability at that time – water on deck (Fig. 3 and 4).

Fig. 3 Recovered Mary Rose[5]

Fig. 4. Recovered Vasa[6]

The first of these ships heeled and sank with all people on board at the beginning of sea battle, before the first shot, after setting the sails and opening the gun-ports. The second heeled and sank at her first parade voyage, having sailed just 1300 m, after sudden blow of wind and taking water through open gun-ports. Additionally, there was a flaw in Vasa design – it had improper proportion of main diameters and location of the gun deck (Litwin, 1995).

To solve that problem, Anthony Deane (1638 – 1721) developed methods to calculate how much armament, ballast and stores should be loaded on ship to bring it to correct waterline. Deane used of ship's plans to evaluate the underwater ship volume, approximating the area of underwater part of frames using area of quarter-circle or rectangles and triangles and summing the areas of frames multiplied by the frame spacing.

The development of infinitesimal calculus became important for practical evaluation of ship stability. Jean Hyacinthe Hocquart (1720) used the equal-width trapezoids to estimate the water-plane areas, while mathematician, Thomas Simpson (1710 – 1761), developed a mathematical rule for numerical approximation of the integral of function, which enabled the practical evaluation of ship's volumes (which means displacement) and volume centers.

These rules (numerical methods), which are still applied using computers, caused dynamic development of stability criteria.

BOUGUER

Pierre Bouguer (1698 – 1758) applied the Hocquart's trapezoid method to estimate areas of water-planes and then the volume of the hull. He developed the "sum of moments method" to compute the center of an object, e.g. the center of ship's buoyancy. Then, Bouguer concluded that this calculus rule can be applied for evaluating the

[4] ibid

[5] http://lisverse.files.wordpress.com/2008/07/maryrose

[6] Wikipedia

integral of functions of one variable or function of several variables, if applied recursively.

To develop hydrostatic theory of ships, Bouguer assumed implicitly the Archimedes principles of hydrostatic that weight and buoyancy of a immersed in water and freely floating body are equal, act in the same vertical line and that they are oppositely oriented (see Appendix, from (A16) to (A18)). He also concluded through geometrical arguments that hydrostatic pressure in the fluid is proportional to the depth and is everywhere "normal" to the surface of submerged body (see Appendix, discussion after (A7) and (A11)).

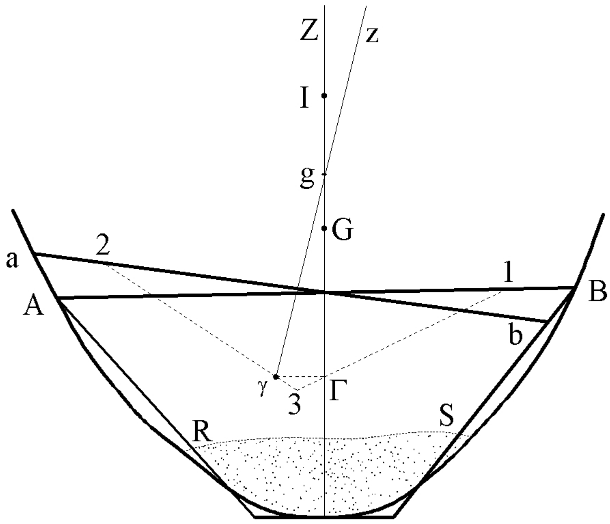

Fig. 5. Cross section of the ship used by Bouguer to determine the metacentre (from Nowacki & Ferreiro, 2003)

Basing on the hydrostatic principles and using the trapezoid rule, Bouguer developed his criterion of stability in the following reasoning (see Fig. 5)[7]:

1. In the upright position the ship's center of mass is in the same vertical line as the center of ship's buoyancy.

2. If the ship changes its position from upright position (waterline A – B), to another position (waterline a – b), resulting in small inclination, the center of buoyancy moves from Γ to γ and the buoyancy force, acting in line Γ – *Z*, changes its line of activity to line γ – *z*. The lines Γ – *Z* and γ – *z* intersect in point *g*, which Bouguer called metacentre.

3. If the center of gravity on line Γ – *Z* lies above, *g* e.g. at position *I*, then the weight of the ship (centered at *I*) is on the inclined side in relation to the center of buoyancy and tends to push the ship further over, causes the ship to be unstable; whereas the center of ship's gravity is below *g*, then the weight, centered in *G*, is on the opposite side of the center of buoyancy and restores the ship to horizontal position.

[7] (Nowacki & Ferreiro, 2003)

Bouguer presented his stability criterion in the following words:

> *"Thus one sees how important it is to know the point of intersection g, which at the same time it serves to give a limit to the height which one can give the center of gravity G, [also] determines the case where the ship maintains its horizontal situation from that where it overturns even in the harbor without being able to sustain itself a single instant. The point g, which one can justly title the metacentre is the term that the height of the center of gravity cannot pass, nor even attain; for if the center of gravity G is at g, the ship will not assume a horizontal position rather than the inclined one; the two positions are then equally indifferent to it: and it will consequently be incapable of righting itself, whenever some outside cause makes it heel over."*

Further, Bouguer also recognized that:

- the metacentric curve for finite angles is the locus of the centers of curvature of the buoyancy centers curve;
- stability improves when ship's beam increases and he claimed that "stability" varies as the cube of beam (as the transverse moment of inertia of waterplane);

and he correctly described how to evaluate the inclining experiment, the idea of Hoste.

EULER

Different approach to derive stability criteria applied Leonard Euler (1707 – 1783). He concentrated on physical and mathematical investigation of the mechanics of ships. Euler derived his stability criterion in the following steps[8]

1. He re-derived the hydrostatic principle of Archimedes from the modern point of view by integration of the hydrostatic pressure distribution in fluid over the surface of the body and stated that:

   > *„Lemma: The pressure which the water exerts on the individual points of a submerged body is normal to the body surface; and the force which any surface element sustains is equal to the same surface element and whose height is equal to the depth of the element under the water surface".*

   It was first mathematical formulation of the hydrostatics.

[8] ibid

2. He determined, by integration of the pressure, the buoyancy force and the center of buoyancy, through which it acts as the volume center of submerged part of the ship and re-confirmed also that in an immersed in water and freely floating body the buoyancy and weight forces must act in the same vertical line and must be equal in magnitude and opposite orientation (in terms of modern hydrostatics presented in Appendix). Euler performed strict integrations for simple shapes to calculate volumes, centers, areas, etc. and did not address numerical evaluation at all.
3. He defined stability criterion in the following form:

   "The stability which a body floating in water in an equilibrium position maintains, shall be assessed by the restoring moment if the body is inclined from equilibrium by an infinitesimally small angle."

   This is the formulation of necessary and sufficient condition of stable ship equilibrium; mathematically written in form: (A16) – equilibrium, and (A19) or (A23) – stable equilibrium.
4. He evaluated the criterion. In the first approach he used a planer cross section of arbitrary shape to examine restoring moments (Fig. 6). He used center of gravity G as the reference point.

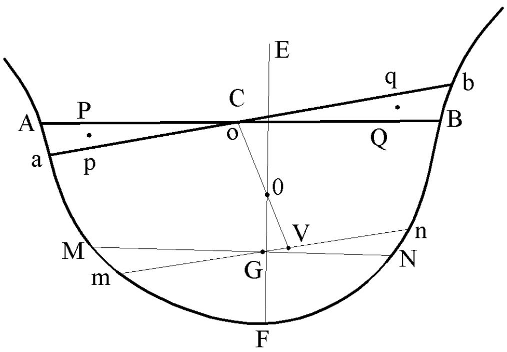

Fig. 6. Cross section of arbitrary shape used by Euler to examine restoring moments (from Nowacki & Ferreiro, 2003)

Euler carried out the following sequence of evaluations (following Nowacki & Ferreiro, 2003) and determined:

- the restoring moment – a couple of forces acting through G and V:

$$M_1 = M\,GV = M\,GO\,dw\,, \quad (1)$$

where O is the center of buoyancy and dw is a small angle, and $M = \gamma(AFB)$ is the buoyancy force M of the cross section before inclination;

- the effect of submerged wedge CBb whose cross section areas is:

$$\frac{BC^2 dw}{2} = \frac{AB^2 dw}{8}\,, \quad (2)$$

and whose restoring moment about G is:

$$M_2 = \gamma \frac{AB^2}{8} dw\left(q_0 + GV\right), \quad (3)$$

where

$$q_0 = \frac{2}{3} Cb = \frac{1}{3} AB\,,$$

- the effect of the emerging wedge ACa in the sense of moment is:

$$M_3 = -\gamma \frac{AB^2}{8} dw\left(p_0 - GV\right), \quad (4)$$

where

$$p_0 = \frac{2}{3} Ca = \frac{1}{3} AB\,,$$

and, finally, by combining all the above moments and replacing γ by $M/(AFB)$, the following restoring moment was obtained:

$$\begin{aligned} M_{rest} &= M_1 + M_2 - M_3 = \\ &= M(GOdw) + \frac{M(AB^2)dw\left(p_0 + q_0\right)}{8AFB} = \\ &= Mdw\left(GO + \frac{AB^3}{12AFB}\right). \end{aligned} \quad (5)$$

Term $\dfrac{AB^3}{12AFB}$ in (5) is, in Bouguer's terminology, the metacentric radius, GO is the distance between center of gravity and center of buoyancy (if G lies above O then GO will reverse its sign). The expression in square brackets – the distance between metacentre and center of gravity is called in modern terminology – the metacentric height; multiplied by dw (small angle) is called in Archimedes nomenclature – the righting arm. Formulae (5) determines the Euler's restoring moment.

Then, Euler derived the formulae determining the restoring moment for three-dimensional floating body of arbitrary shape. It is similar to formula (6):

$$M_{rest} = \gamma V dw\left(GO + OM\right) = \gamma V dw(GM), \quad (6)$$

where $OM = I_T/V$ is the metacentric radius, I_T is the planar moment of inertia of the waterplane (waterline is described by function $y = y(x)$), and V is the volume occupied by submerged part of the body. The expression in brackets is the metacentric height GM.

Euler used the initial restoring moment as the stability criterion and he discussed the implications of the change of parameters included in formulae (6), for example, the stability of the ship is improved if center of gravity *G* is lowered, center of buoyancy *O* is raised and/or the beam of the ship is widen.

SHIP STABILITY CAPACITY

The metacentric height *GM* was used at least a hundred years to evaluate the ship stability. *GM* is important indicator of initial stability, however, it does not poses information on the “stability capacity of the ship”. This is incorporated in the curves of stability righting arm, which is shown in (Fig. 7) versus heeling angels φ.

Good example of the importance of the ship stability capacity is sinking of the naval ship *Captain* in Biscay Bay, in 1870, in heavy sea state, the case well known from the literature; *GM* of the ship was equal to 0.79m. This ship was accompanied by another naval ship *Monarch*, similar in dimensions and features; *GM* of the ship was equal to 0.73m. *Monarch* survived this sea state. For naval architects it was a surprise. The explanation of this case was simple: the freeboard of *Captain* was equal to 1,98m, while freeboard of *Monarch* was equal to 4.27m and the righting arm curves of *Monarch* had much better parameters then the *Captain* arms. The conclusion was self-evident – the stability parameters of heeled ship should be also examined; initial stability alone is not sufficient as a stability measure. (Today, this conclusion is expressed by conditions (A25).)

This conclusion was not an original one. Atwood, 1796, was the first who started to investigate the stability properties of homogeneous solids of simple shape at large angles of heel and he concluded that stability must be judged over a range of heel angles. Atwood and Vial du Clairbois introduced the term *GZ* for the “righting arm”[9].

The capsizing of *Captain* caused that at the end of XIX century the curves of righting arms, popularized by admiral Reed, were universally applied.

In 1939, Rahola published his doctoral thesis (Rahola, 1939) based on extensive statistics and analysis of the stable and unstable vessels and then he proposed a stability criterion referring to:

- the metacentric height (initial stability) – he determined the height (number) as lower limit, and on
- the curve of righting arm – he determined the limit angle of the curve.

[9] ibid

In terms used above, Rahola formulated the “stability capacity” of ship being at rest in calm water. His proposal was used as the basis of stability regulations of national and international authorities (e.g. IMO). The example of such regulations, according to Polish Register of Shipping (PRS, 2010), are presented in Fig. 7.

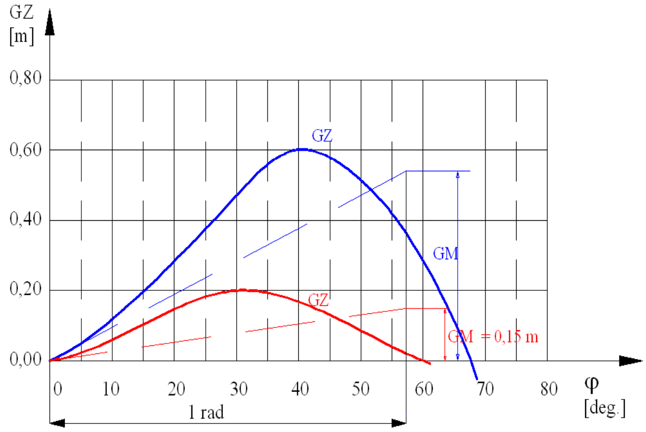

Fig. 7. The example of the criteria acc. to PRS (red color) and actual *GM* and righting arm curve of a typical general cargo ship (blue color)

SHIP STABILITY IN DAMAGE CONDITION

The discussed criteria refer to the intact ship stability. The stability for damaged ships was firstly standardized by metacentric height *GM* and the free board (IMO, 1974) and then by the righting arm curve for damaged ships (IMO, 1990). The problem is not easy to solve as the buoyancy center moves in damage ship condition and determination of the righting arm curve in the terms of intact stability is not unique; to do that the knowledge of principal axes of inertia for actual damage water-plane is needed (Pawłowski, 2002).

It should be mentioned that, as already reported by Marco Polo in XIII century, that Chinese junks featured watertight subdivision – noting the problem of damage stability, which elsewhere was addressed six centuries later. However, it is not known whether they used any criteria but China was definitely a forerunner in this respect.

WATER ON DECK

The problem of water on deck – which affected naval ships with large openings in their sides (gun-ports) in sixteen and seventeen centuries, has again dramatically come back for Ro-Ro vessels in damage conditions. It is enough to mention the tragic capsizing of the passenger ro-ro ships: *European Gateway* (1982), the *Herald of Free Enterprise* (1987), and *Estonia* (1995)). Fig. 8. presents two slides from the simulation of passenger Ro-Ro vessel motion in waves in damage condition.

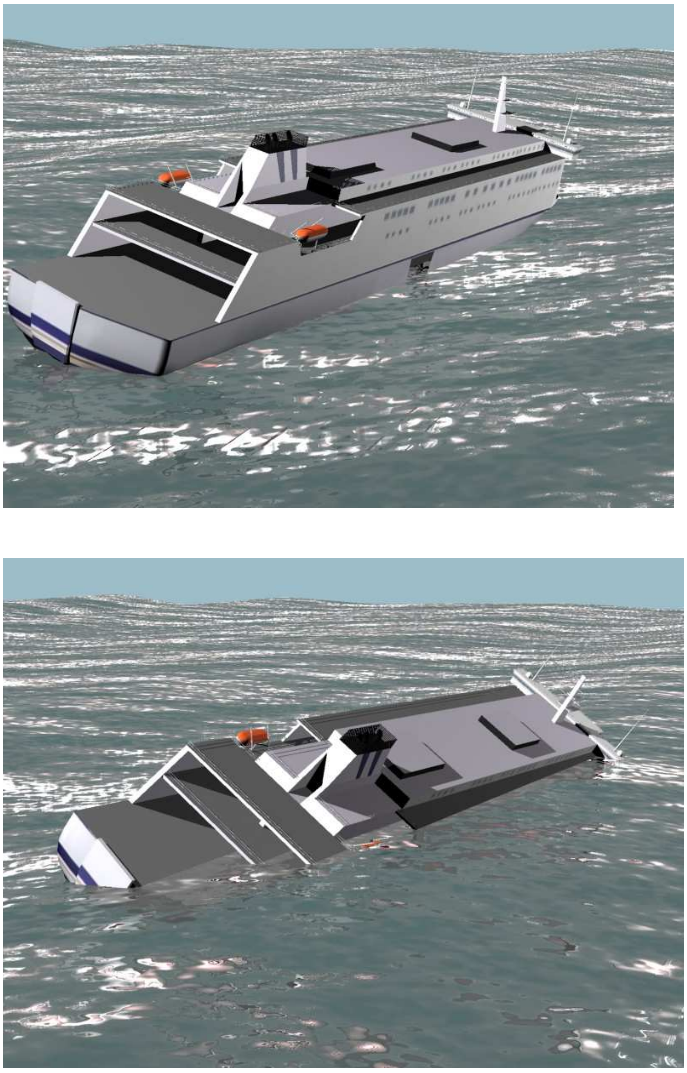

Fig. 8. Simulation of passenger ferry motion in wave in damage condition

IMO has adopted in 1997 the criterion, s-factor, which was modified in 2009. The criterion has the following form:

$$s = C(25/48\mathrm{GZmaxRange})^{1/4}, \qquad (7)$$

where the factor s is the probability of collision survival, the coefficient C accounts for the effect of the final angle of equilibrium, with $C = 1$, if the final angle of equilibrium $\phi_e \leq 25^\circ$, $C = 0$, if $\phi_e > 30^\circ$, and $C = [(30-\phi_e)/5]^{1/2}$, otherwise. GZ_{max} is the maximum righting lever (in meters) within the *range* as given below but not more than 0.1 m. *Range* is understood as the range of positive righting arms curve beyond the angle of equilibrium but not more than 20˚, and not more than the angle of immersion of non-weathertight openings.

However, criterion (7) reflects the so-called good engineering judgment and, therefore, there are attempts to develop rational and more accurate damage stability criterion to predict the capsizal resistance of damaged Ro/Ro vessels. The so called static equivalent method (SEM) has been developed (Vasalos *et al.*,1996, Pawłowski, 2004). The SEM for Ro/Ro ships postulates that the ship capsizes in a way that is quasi-static and is based on the heeling moment caused by the elevated water on the vehicle deck. This method was developed following observations of the behavior of damaged ship models in waves. The most important observations are:

- When the heeled ship reaches the point of no return (PNR) it behaves quasi-statically, with marginal transverse stability and very subdued roll motions.
- The PNR (the critical heel) generally occurs at an angle very close to ϕ_{max}, the angle where the static *GZ* curve for the damaged ship reaches its maximum.
- The critical amount of water on the vehicle deck can be predicted from static calculations by flooding the undamaged vehicle deck until the heel angle reaches ϕ_{max}.
- The critical and unique measure of the ship's survival capability is the level h that this critical amount of water is elevated above the sea level at the point of no return, as shown in Fig. 9.

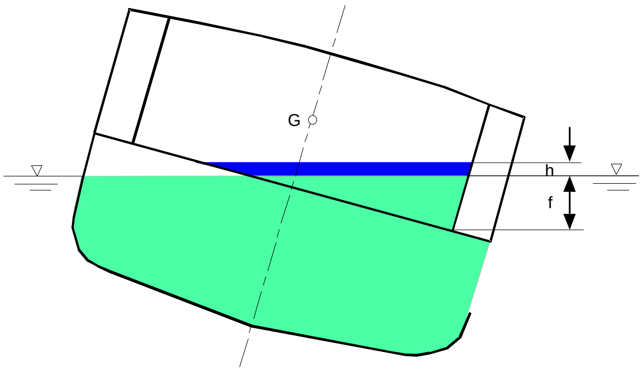

Fig. 9. A damaged Ro/Ro ship with a rise of water on the car deck at the PNR

- The model tests and subsequent simulations indicated that this elevation h of water on deck could be directly linked to the sea state (determined by the significant wave height H_s).

The critical heel angle (PNR) is understood as the heel angle induced by the elevated water h on deck at which the equilibrium of the ship is unstable; this angle is crucial for the SEM, as the elevation of water is calculated just at that angle, which in turn defines the critical H_s. The sought boundary stability curve takes the form:

$$h = 0.085\, H_s^{\;1.3}, \qquad (8)$$

where H_s determines the median sea state the ship can withstand with given stability. The critical wave height H_s depends solely on the elevation of water at the critical heel angle. Knowing the critical sea state H_s from equation (8) for a given damage case, the factor s (probability of collision survival) can be readily obtained from the distribution of sea states occurring at the moment of collision (Fig 10).

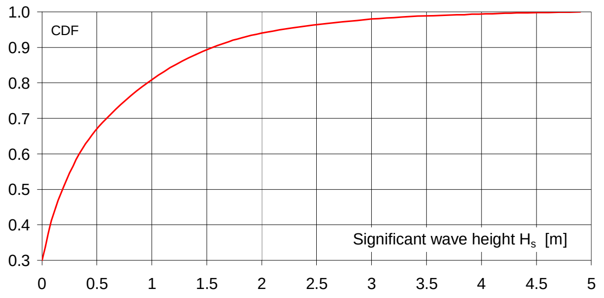

Fig. 10. IMO distribution of sea states occurring at the moment of collision

The probability of collision survival equals simply to the probability that the critical significant wave height H_s is not exceeded at the moment of collision. Thus, the factor s equals CDF for given H_s. Using the sea state distribution proposed by IMO the s-factor is approximated by the following formulae:

$$s = (0.7494\,x^3 - 2.4095\,x^2 + 2.6301\,x + 0.0148)^{1/3}, \qquad (9)$$

where $x = H_s/4$ is in meters. For $H_s > 5$ m, $s = 1$.

Criteria (7), (8) and (9) are the first ones which take into account the randomness of sea waving.

SMALL VESSELS

The statistics on smaller vessel losses due to capsizing in high sea states raised the problem of safety of these vessels, particularly fishing vessels. The problems in stability standardization of smaller vessels are as follows:

- unfavorable relation between stability capability and the magnitude of external heeling moments caused by waves (Fig. 11);
- shift of weight on smaller ships causes more significant change of gravity center and often significant change of stability;
- water on deck often dramatically reduces the righting arm (Fig. 12);
- inadequate stability criteria and standards – based on static stability in calm water, and not on real dynamic behavior in waves.

Therefore, there is a urgent need of development the rational criteria, preventing capsizing of smaller ships, especially fishing vessels, basing on their dynamic behavior in waves.

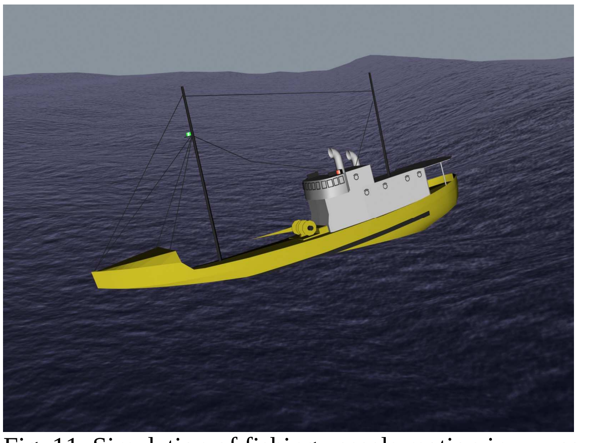

Fig. 11. Simulation of fishing vessels motion in waves

One significant element is water trapped on vessel deck (Fig. 12).

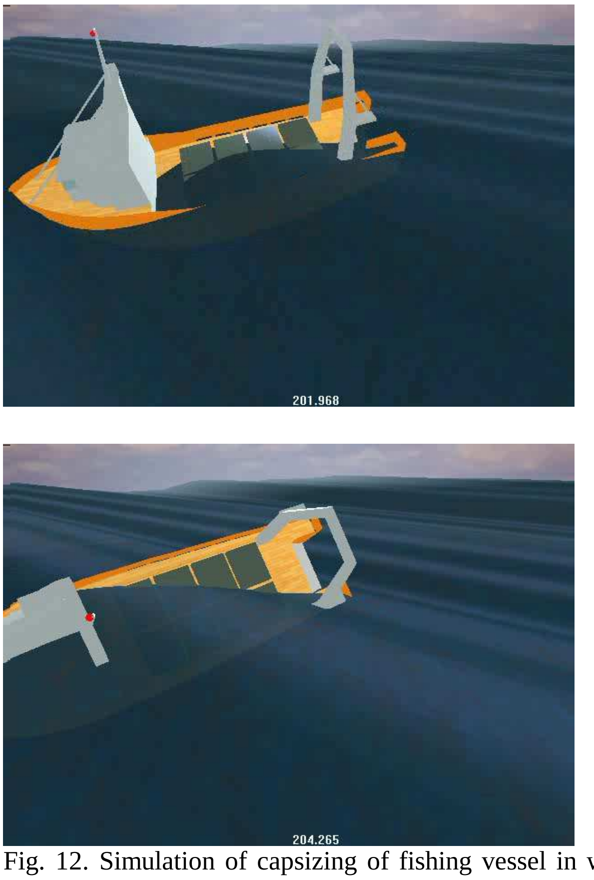

Fig. 12. Simulation of capsizing of fishing vessel in waves due to the water trapped on deck

STRICT METHOD OF MODERN HYDROMECHANICS

It is interesting to show how the static stability problems are expressed in terms of the modern hydrostatics. Let us consider a special body shape placed in water – a cylinder of parabolic cross section of homogenous material (Fig. 13).

To evaluate hydrostatic force and restoring moment induced by a body being at rest in water and inclined to angle $\Phi = 0.2$ rad formulae (A12) and (A13), presented in Appendix, are used.

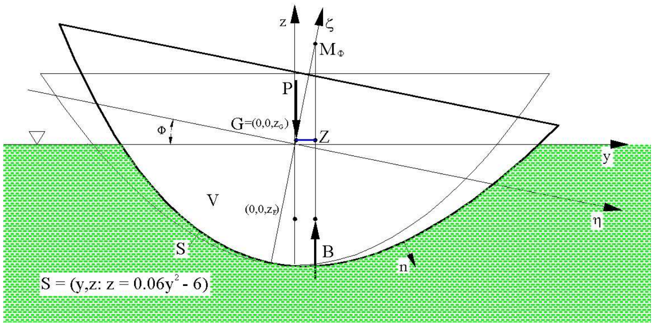

Fig. 13. Forces and restoring moment acting on inclined cylinder of parabolic cross section

The results of integrations, after transforming the domain of integration from surface S to volume V occupied by the body, using the Gauss theorem, are as follows:

$$M_x = \rho g \int_S z\left[\left(y - y_g\right)n_3 - \left(z - z_g\right)n_2\right]ds =$$

$$= \rho g \int_V \left(y - y_g\right)dv =$$

$$= \rho g \left[\int_V J\left(\eta\cos\Phi + (\zeta - \zeta_g)\sin\Phi\right)dv\right] = \qquad (10)$$

$$= \rho g \cos\Phi \int_d^e \eta \int_{0.06\eta^2-6+c}^{a\eta} d\zeta\, d\eta +$$

$$+ \rho g \left[\sin\Phi \int_d^e \int_{0.06\eta^2-6+c}^{a\eta} (\zeta - \zeta_{g)} d\zeta\, d\eta\right],$$

where Jacobian J = 1, $a = \tan \Phi$, and c – results from (A 16), requiring that buoyancy $\boldsymbol{B}$ should be equal to weight $\boldsymbol{P}$ after inclination of the body to angle Φ, and d, e are the abscises of intersection of points of parabola and waterline. For a body inclined, for example to the angle Φ = 0.2 rad, the righting arm obtained from (10) is:

$$GZ = |M_x / \rho g V| = 1.06903\,.$$

The same evaluation carried out applying directly formula (A12) and (A13) (without transforming to other domain) and performing the integrations with the use of the numerical methods yields:

$$GZ = |M_x / \rho g V| = 1.01413.$$

To perform the integrations numerically cross section S of the body was divided into 300 segments.

The initial stability of the ship can be easily evaluated using the following approach:

- The position of inclined body, let say to the angle Φ = 0.2 rad, can be described using the following formulae:

$$z = z_0 + \Phi y_0$$
$$p = -\rho g\left(z_0 + \Phi y_0\right), \qquad (11$$

  where $(y_0, z_0) \in S$ in upright body position;

- Substituting (11) to (A13), the following integral is obtained:

$$M_x = \rho g \int_S z_0\left[y_0 n_3 - \left(z_0 - z_g\right)n_2\right]ds +$$
$$+ \rho g \Phi \int_S y_0\left[y_0 n_3 - \left(z_0 - z_g\right)n_2\right]ds. \qquad (12)$$

The first integral – determining the restoring moment of the body in upright position, is equal to 0. Transforming the domain of integration from surface to S to volume V occupied by the body, using the Gauss theorem, the following formulae is obtained:

$$M_x = \rho g \Phi \int_S y_0\left[y_0 n_3 - \left(z_0 - z_g\right)n_2\right]ds =$$

$$= -\rho g \Phi \left[\int_V \left(z_0 - z_g\right)dv + \int_{-b}^{b} y^2 dy\right] =$$

$$= -\rho g \Phi\left[\left(z_V - z_g\right)V + I_x\right] = \qquad (13)$$

$$= -\rho g \Phi\left[\left(z_V - z_g\right) + I_x / V\right]V =$$

$$= -\rho g \Phi\left[\left(z_V - z_g\right) + r_0\right]V =$$

$$= -\rho g \Phi |GM| V = -\rho g |GZ| V$$

Where $r_0 = I_{xx}/V$ is the metacentric radius, I_{xx} is the inertial moment of the waterplane in relation to axis X, $r_o + (z_V + z_G) = |GM|$, and $\rho g\ V$ is the buoyancy force. For the case considered $I_{xx} = B^3L/12$, B is the breadth of the body on the water level, and L is the length of the body. Performing the integrations of the second integral the righting arm is obtained:

$$GZ = \Phi\left(\left(z_V - z_g\right) + I_{xx} / V\right) = 0.988667\,.$$

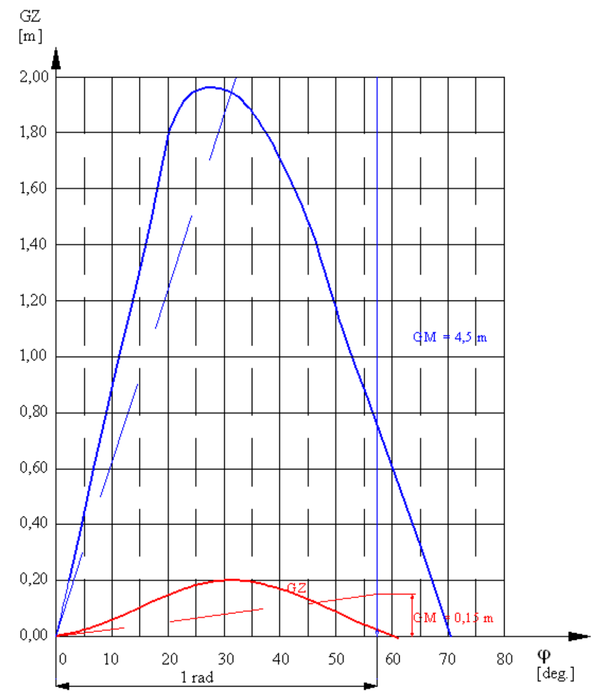

Fig. 14. Righting arm curve of cylinder of parabolic cross section

The difference between the results obtained from the three methods is less than 0.09.

However, for larger angles of heeling and practical shapes of ships, described geometrically or numerically, the only numerical method for integrating (A13) can be applied. For example the heeling arm curve computed numerically for the case considered is presented on Fig 14.

It is interesting to evaluate the stability of inclined body with water on deck. The hole in body's side (Fig. 15) can be interpreted as damage hole in Ro-Ro passenger vessel or gun-port in naval ships of XVI/XVII century. The water on deck dramatically reduces the righting arm of the body (Fig. 16). Formulae (13) shows that the initial stability depends on the third power of ship breadth *B* of the body on the sea level, and is reduced according to this rule when water on deck decreases the breadth, what explains the problem of "water on deck" in the initial angles of ship heeling. The righting arm curve (Fig 16) shows that the body with hole over the first deck (small freeboard) has a very little "stability capacity" comparing with the "stability capacity" of the body without the hole (high freeboard) – (Fig. 14).

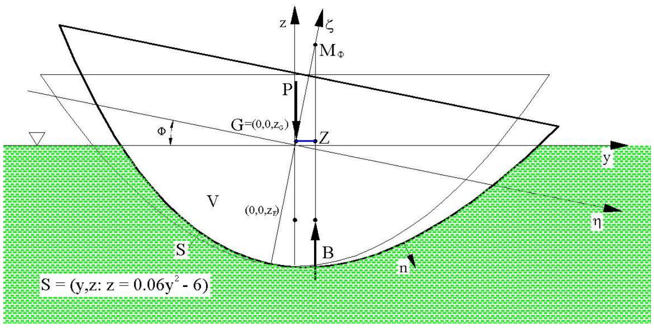

Fig. 15. Forces and restoring moment acting on inclined cylinder of parabolic cross section with water on deck

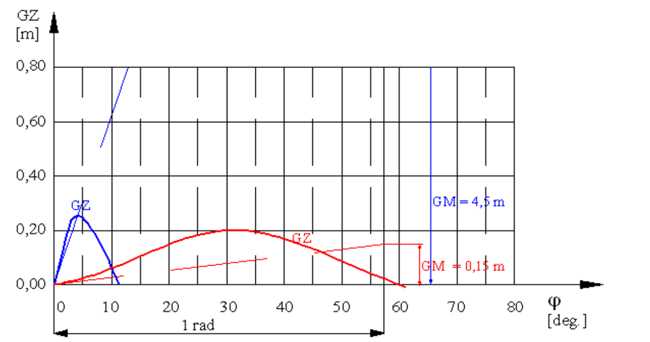

Fig. 16. Righting arm curve of cylinder of parabolic cross section with water on deck which is 0.4m over the sea level

SUMMARY AND CONCLUSIONS

The development of safety criteria for ship's stability started more than 22 centuries ago. Archimedes was the first who developed a model showing the mechanics of ship's stability – he developed a stability measure similar to the righting arm.

The Archimedes ideas were used 18 centuries later to solve the problem of stability of the ship being at rest, placed in water also being at rest. New ideas and notions have been developed and introduced step by step:

- Stevin introduced the concept of hydrostatic pressure distribution proportional to the weight of a water prism above the depth;
- Host attempted to quantify the problem of ship's stability;
- La Criox correctly recognized the role of weight and buoyancy forces acting in the same vertical line at equilibrium.
- Hocquart used the trapezoidal rule to estimate the water plane areas and Simpson developed a quadrature rule for planar curves, which yields numerical approximation of function integral.

The numerical methods developed enabled computations of areas, volumes and static moments and caused rapid development of ship's stability criteria. It should be emphasized that numerical methods are still the basic tool in the naval architecture as the ship's shape is described geometrically – by body lines or in the form of its numerical representation.

First who applied numerical methods to develop stability criterion was Bouguer. Through the geometrical considerations and numerical computations he began formulating in 1732 the concept of metcenter which he used as the stability criterion. Bouguer stated that: *the ship's stability depends on the position of the ship's mass center in relation to the metcenter* and used this as an stability criterion.

Three years later, Euler derived mathematical form of restoring moment and used it as the criterion saying that: *ships should have positive restoring moment.*

The Bouguer and Euler criteria refer to the initial ship's stability (for small angles of heeling). *Metacentric height GM was used at least a hundred years to evaluate the ship stability.* However, *GM* is important indicator of initial stability and refers to as the necessary condition of the ship stability and does not poses information on the "stability capacity of the ship", which is incorporated in the curves of stability righting arm. After accidences of ships due to the loss of stability in large angles of heel additional *criterion in form of the limiting curve of righting arm* have been set.

The discussed criteria refer to the intact ship stability. The stability for damaged ships was firstly standardized by metacentric height *GM* and the free board and then by the righting arm curve for damaged ships. However, the problem is not easy for solving as the righting arm curve should be determined in ship's principal axes of inertia for actual damage waterplane.

Existing of the problem of damage stability was well understood in China. The watertight subdivision was already used as early as, at least, XIII century but it is not known whether any criteria for damage stability were applied.

The problem of water on deck which affected naval ships with gun-ports in sixteen century, has again appeared for Ro-Ro vessels in damage conditions. The mechanism of reduction of initial vessl's stability explains formula (14).

In 1997, *IMO has adopted criterion, modified in 2009, determining the probability of collision survival*. However, there are still attempts to develop rational and more accurate damage stability criterion to predict the capsizal resistance of damaged Ro/Ro vessels. The so called *static equivalent method (SEM) has been developed* as it is presented above.

Developed criteria for Ro-Ro vessels in damage conditions are the first ones which take into account the randomness of sea waving.

The stability criteria for ships in rest cannot be applied to smaller vessels like the fishing ones, sailing in waves. In this case the development of stability criteria should be based on the equations of ship motions in waves. The first results obtained has shown that the underwater shape of the vessel, changing in time, distribution of mass, vessel speed and many other parameters influence its dynamic stability (the capsizal resistance).

Sea waving is a stochastic process, which additionally complicates solving of the problem of dynamic stability of small vessel moving in waves in the sense of the criterion. Still, it is a challenge for researchers.

The history of development of ship's stability criteria seems to be, on one hand, never ending story and, on the other hand, it reflects a human life pursuing the perfection.

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APPENDIX

HYDROSTATICS (Krężelewski, 1982)

The reference system is defined in Fig. A1.

To determine the fluid reaction on a submerged body the following unit pressure force – pressure vector ***p*** is defined:

$$\mathbf{p}_n = \lim_{\Delta s \to 0} \frac{\Delta \mathbf{R}}{\Delta s}, \quad \text{(A1)}$$

where ΔS is a surface element, and $\Delta \boldsymbol{R}$ is the force distributed on ΔS.

If the pressure vector is known then the reaction ***R*** on entire submerge surface *S* is determined by the following integral:

$$\mathbf{R} = \int_S \mathbf{p}_n ds, \quad \text{(A2)}$$

and moment ***M***:

$$\mathbf{M} = \int_S \mathbf{r} \times \mathbf{p}_n ds, \quad \text{(A3)}$$

where ***r*** is the radius vector of surface *S* in relation to the pole – normally the center of mass *G* is chosen.

It turns out that:

$$\mathbf{p}_n = \mathbf{n} P, \quad \text{(A4)}$$

where ***n*** is the normal to surface *S* vector, oriented into the fluid, and *P*, called the stress tensor in the fluid, has the following form:

$$P = \begin{bmatrix} p_{xx} & p_{xy} & p_{xz} \\ p_{yx} & p_{yy} & p_{yz} \\ p_{zx} & p_{zy} & p_{zz} \end{bmatrix}. \quad \text{(A5)}$$

Stress tensor depends only on point of fluid domain while pressure vector $\boldsymbol{p}_n$ depends on point of fluid and on orientation of element ΔS in the space.

The mechanics of fluid is described by the following Navier–Stokes equations and the equation of mass conservation:

$$\frac{d\mathbf{v}}{dt} = \mathbf{F} + \frac{1}{\rho} divP, \qquad \frac{d\rho}{dt} + \rho div\mathbf{v} = 0 \quad \text{(A6)}$$

where ***F*** is the unit mass force, ρ is the fluid density, and ***v*** is the fluid velocity field in the fluid. These equations describe the velocity and pressure fields in the fluid.

Assuming that the water and the body are at rest, $\boldsymbol{v} = 0$, and that the water is incompressible fluid, $\rho = const$, then (A5) assumes the following form:

$$P = -\begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix}, \quad \text{(A7)}$$

as there are not tangential stresses in water for $\boldsymbol{v}=0$ – only the stresses on the tensor diagonal have values different then zero and they are equal. It can be concluded that in such a case (no tangential stresses) the pressure in water does not depend on the orientation of the surface element ΔS, on which the pressure is acting. In the history of ship criteria development this pressure feature was express as the pressure is “normal”.

Under the above assumptions equation (A6) is reduced to the following form:

$$\mathbf{F} - \frac{1}{\rho} grad(p) = 0 . \quad \text{(A8)}$$

(A8) is a vector equation, saying that mass force ***F*** is parallel to the gradient of pressure.

Water is in the gravitational field, therefore force $\boldsymbol{F} = -grad\ U$ has a potential U=g*z* and: $\boldsymbol{F} = -(0, 0, g)$. is parallel to the vertical axis *Oz*. It implies, basing on (A8), that *grad U* is parallel to ***F*** and that surfaces of constant (isobaric) pressure are horizontal planes $z = 0$.

Taking that into account equation (A8) takes the following form:

$$grad\left(U + \frac{p}{\rho}\right) = 0, \quad \text{(A9)}$$

and its solution is as follows:

$$gz + \frac{p}{\rho} = C, \quad \text{(A10)}$$

where constant $C = p_a/\rho$ is determined from the condition on free surface $z = 0$, and p_a is the atmospheric pressure, so, the solution is:

$$p = p_a - \rho g z, \quad \text{(A11)}$$

This formulae also implies that the surfaces of constant pressure are horizontal planes, parallel to the free surface plane $z = 0$.

Accounting (A2) and (A3), water reaction – ***R*** and moment – ***M*** to the placed in water ship is:

$$\mathbf{R} = -\int_S p\mathbf{n}ds \tag{A12}$$

$$\mathbf{M} = -\int_S \mathbf{r}\times\mathbf{n}pds\,, \tag{A13}$$

Applying Gauss theorem to (A12) water reaction – ***R*** on placed vessel in water takes the following form:

$$\mathbf{R} = -\int_V gradpdV\,, \tag{A14}$$

which after substituting (A11) yields:

$$\mathbf{R} = (0,0,\rho gV) \equiv \mathbf{B}\,, \tag{A15}$$

where the hydrostatic reaction ***R*** is called in the shipbuilding the buoyancy force and is denoted by ***B***, and *V* is the vessel displacement (domain occupied by vessel in water). It can be seen that the buoyancy force ***B*** is equal to the weight of water displaced by the ship, which value is equal to ρgV, that acts vertically and is oriented upwards. In Archimedes words it sounds: ***B*** *is equal to the weight of water displaced*.

CONDITIONS OF SHIP EQUILIBRIUM

The equations of ship motions being at rest in the water at rest are as follows:

$$\sum_i \mathbf{F}_i = 0 \text{ and } \sum_i \mathbf{M}_i = 0\,, \tag{A16}$$

where $\Sigma \boldsymbol{F}_i$ and $\Sigma \boldsymbol{M}_i$, $i = 1,\ldots, n$, are sums of external forces and moments acting on the vessel. (A16) are the necessary conditions of vessel equilibrium in relation to the calm free surface of the sea.

If the only external forces acting on the vessel are $\boldsymbol{F}_1 = \boldsymbol{B}$ and $\boldsymbol{F}_2 = \boldsymbol{P}$, where ***P*** is the weight of the vessel, acting in line parallel to axis *0z* and oriented downwards then in equilibrium (condition (A16) – external forces must be equal) magnitude of ***B*** is equal to the magnitude of ***P***. In Archimedes words this equilibrium condition sounds: *Any solid lighter then fluid will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of fluid displaced*.

The line of activity of:

- the weight ***P*** passes through the center of mass *G* and
- the buoyancy force ***B*** passes through the center of buoyancy $C_{B,}$

and their position radiuses in the reference system *O* are r_G and r_{CB}, respectively. The second necessary condition (A16) of equilibrium (moments must be equal):

$$\mathbf{r}_G \times \mathbf{P} + \mathbf{r}_{CB} \times \mathbf{B} = 0 \tag{A17}$$

implies that $\boldsymbol{r}_G = \boldsymbol{r}_{CB}$ and that the vertical lines of ***P*** and ***B*** activity covers; the forces act in the same line. If the pole is center of mass *G*, then $\boldsymbol{r}_G = 0$ and equation (A17) becomes simpler:

$$\mathbf{r}_{CB} \times \mathbf{B} = 0\,,$$

what means that $\boldsymbol{r}_{CB} = 0$ and the lines of ***P*** and ***B*** activity covers.

Concluding: If the only external forces acting on the vessel are buoyancy ***B*,** acting trough the $C_{\boldsymbol{B}}$, and weight ***P***, acting trough the *G*, then in the equilibrium state these forces act in the same vertical line, they have the same magnitude and they are oppositely oriented.

The equilibrium can be stable, neutral and unstable. The vessel is stable if being inclined from the equilibrium position has ability to come beck to equilibrium, after removing the reason of inclination. It means that in case of stable equilibrium, restoring forces $\boldsymbol{F}_s$ and restoring moments $\boldsymbol{M}_s$ must appear to restore the vessel to the equilibrium and they should have signs opposite to the inclinations, what can be written in the following form:

$$\frac{\partial \mathbf{F}_s}{\partial s} < 0 \text{ and } \frac{\partial \mathbf{M}_s}{\partial s} < 0 \tag{A18}$$

Conditions (A18) are sufficient conditions for stable equilibrium. The vessel’s equilibrium is neutral for its surge, sway and yow, and stable for heave, what can be easily proved. The real problem is with transfer stability of the vessel inclined to the angle Φ. The sufficient condition for this inclination reads ($s = \Phi$):

$$\frac{\partial \mathbf{M}_\Phi}{\partial \Phi} < 0\,, \tag{A19}$$

where $\boldsymbol{M}_\Phi$ is the vessel’s transverse restoring moment.

Let the vessel of weight ***P***, being in equilibrium in upright position ($\Phi = 0$), is slowly listed to angle Φ. So, to fulfill necessary condition (A16), displacement *V* need to be constant (*V* = *const*). However, the shape of the vessel’s underwater part and, therefore, its center (center of buoyancy) $C_{B\Phi}$, is changing as it is shown in (Fig. A1).

The directions of weight ***P*** and buoyancy forces ***B*** are perpendicular to the new water-plane and therefore they are parallel. The distance between these two forces, *GZ*, is called the arm of static stability.

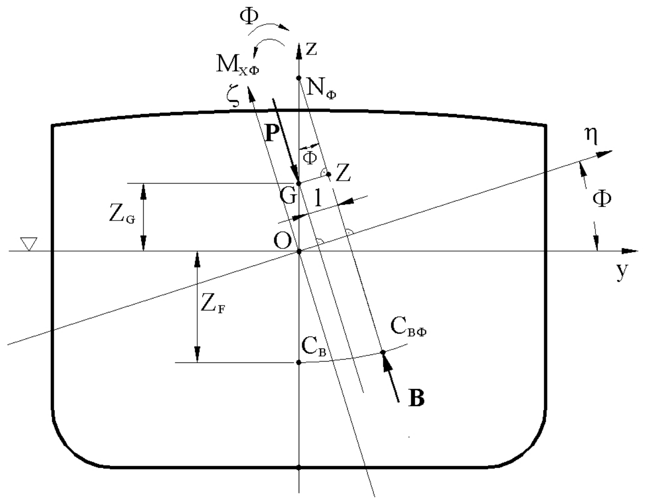

Fig. A1. Restoring moment generated by the listed vessel

Restoring moment $\boldsymbol{M_\Phi}$ in relation to the mass center C_G is equal to:

$$\left|M_\Phi\right| = \left|\overline{G}\,N_\Phi \times \mathbf{B}\right| = -\overline{GN}_\Phi \cdot \mathrm{B}\sin\Phi = -\mathrm{B}l \quad \text{(A20)}$$

as M_Φ acts oppositely to Φ, and B is the magnitude of vector $\boldsymbol{B}$. $GZ = \overline{GN}_\Phi \sin\Phi \equiv l$ is the arm of static stability which depends on the shape of underwater part of the vessel, on mass distribution on the vessel and heeling angle $\boldsymbol{\Phi}$. Arm GZ is known when $C_{B\Phi}$ and N_Φ are known.

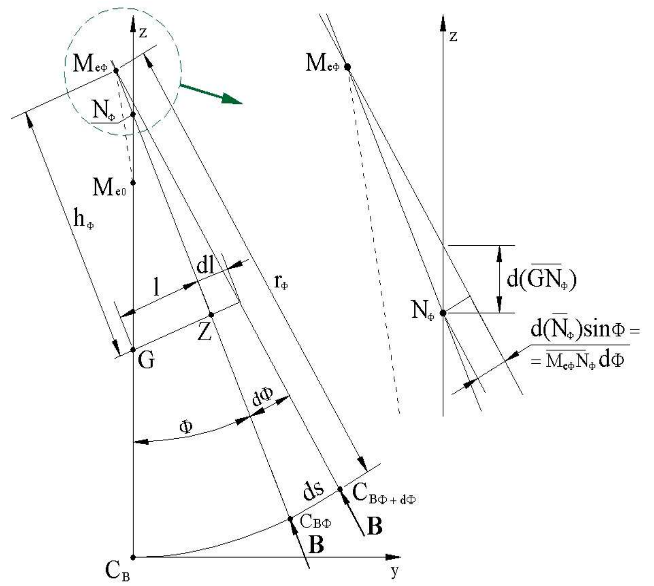

Fig. A2. Definition of the radius of the curve of buoyancy centers $C_{B\Phi}$

The radius r_Φ of the curve of buoyancy centers $C_{B\Phi}$ is defined as follows (Fig. A2):

$$r_\Phi = \lim_{\Delta\Phi\to 0}\frac{\Delta s}{\Delta\Phi} \quad \text{(A21)}$$

The center of this curve – $\boldsymbol{M_{e\Phi}}$, and its radius – r_Φ, are called in shipbuilding the metacentre and metacentric radius, respectively. For initial angles of heeling $M_{e\Phi} \rightarrow M_{e0}$ and for such vessel's position GM_{e0} is called the metacentric height. For small vessel's heeling ($\sin\Phi \approx \Phi$) and condition (A19) takes the following form:

$$\frac{\partial M_\Phi}{\partial\Phi} = -\mathrm{B}\left(\frac{\partial \overline{GN}_\Phi}{\partial\Phi}\sin\Phi + \overline{GN}_\Phi\cos\Phi\right) = -\mathrm{B}\frac{\partial l}{\partial\Phi} < 0 \quad \text{(A22)}$$

thus

$$\frac{\partial \overline{GN}_\Phi}{\partial\Phi}\sin\Phi + \overline{GN}_\Phi\cos\Phi = \frac{\partial l}{\partial\Phi} = h_\Phi, \quad \text{(A23)}$$

This inequality says that heeled vessel to angle $\boldsymbol{\Phi}$ is stable when metacentric height h_Φ has positive value.

Concluding, the necessary and sufficient conditions of the transverse stability of a floating vessel without heeling are:

$$\mathbf{B} = \mathbf{P}, \quad \mathbf{M}_\Phi = 0 \text{ and } GM_{e0} > 0, \quad \text{(A24)}$$

and for heeled vessel to angle $\boldsymbol{\Phi}$:

$$\mathbf{B} = \mathbf{P}, \quad \text{and} \quad h_\Phi > 0, \quad \text{(A25)}$$

Fig. A3 shows the curve of stability arm l floating with heeled position $\boldsymbol{\Phi}_0$. In this case GM_{e0} <0 and angle $\Phi = 0$ is not the angle of equilibrium. For $\boldsymbol{\Phi} = \boldsymbol{\Phi}_0$, $GM_{\Phi 0}=(dl/d\Phi)_{\Phi=\Phi 0}$ >=0 and the equilibrium of the heeled vessel is stable.

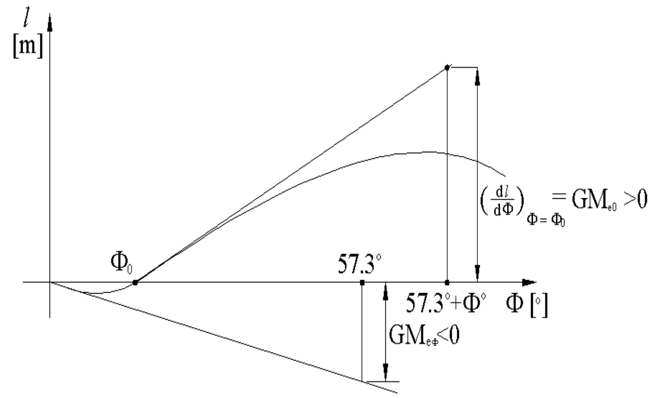

Fig. A3. Curve of righting arm