Stability Assessment of Small Vessels

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Abstract

Small vessels such as boats and fishing cutters often lack technical documentation. Therefore, it is necessary to develop a method for stability assessment of such boats based on a criterion that does not require the reconstruction of the vessel's documentation. This can be achieved on the basis of the inclining test.

1. Assumptions

Formally, small vessels are not covered by stability requirements. IMO regulations apply to vessels over 24 m in length [2]. Statistical data over the years invariably indicate a large number of casualties in the fishing fleet [3], of the order of 24 thousand of life losses per year. Thus, Polish Register of Shipping (PRS) has embarked on the development of a stability criterion for small vessels. The criterion shall be based on the following assumptions:

- the vessel lacks technical documentation;
- the heeling moment is generated by people and gear located on the vessel's side;
- the action of the moment is static;
- for boats without a deck the heeling moment cannot induce a heel exceeding the range of initial stability ϕ_D, i.e. the angle at which the side edge is immersed in the water or the (assumed) bilge comes out of the water, whichever is smaller.

2. Theoretical basis

A heel of the vessel due to the action of a certain nominal weight p, shifted from the ship's plane of symmetry (PS) to the side results from the equilibrium between the heeling and restoring moments. The heeling moment, induced by a horizontal shift of the weight is equal to $pe\cos\phi$, where e is the horizontal shift of the weight and ϕ is the heeling angle. If we assume that the metacentre remains constant, the righting moment is given by the equation $Dh_0\sin\phi$, where D is the vessel's buoyancy (unknown), and $h_0 \equiv GM$ is the initial metacentric height (unknown). By equating the two moments we obtain an equilibrium equation:

$$pe\cos\phi = Dh_0\sin\phi,$$

yielding the heel angle:

$$\mathrm{tg}\phi = \frac{pe}{Dh_0}, \tag{1}$$

called the *metacentric* equation. This angle should not exceed the range of initial stability, denoted by ϕ_D, i.e. $\phi \le \phi_D$, where $\tan\phi_D = 2f/B_m$; f is the minimum freeboard $H - T$, but not more than $0.8T$, H is the depth of the boat, T is the draft, and B_m is the maximum breadth of the boat. Hence, we get the following inequality:

$$\frac{pe}{Dh_0} \le \frac{f}{\frac{1}{2}B_m}, \tag{2}$$

The product of buoyancy and metacentric height Dh_0, termed in ship's theory the *coefficient of stiffness* is important not only in ship statics but also in ship dynamics, as it determines the vessel's natural frequency of roll. Substituting $e = ½B_m$, equation (2) yields:

$$(Dh_0)f/(½B_m)^2 \ge p.$$

Dividing this inequality by the gravitational acceleration g, we get an inequality in the category of mass:

$$(Mh_0)f/(½B_m)^2 \ge m, \tag{3}$$

where m is the mass of the nominal weight, and M is the vessel mass. Inequality (3) can be regarded as a stability criterion provided the mass stiffness coefficient Mh_0 is known and the nominal mass m is defined.

Introducing the notation $K \equiv Mh_0$ for the coefficient of mass stiffness, inequality (3) reads:

$$Kf \ge m(½B_m)^2. \tag{4}$$

It can also be written as: $Kf/(½B_m) \ge m(½B_m)$. The latter inequality indicates that its left hand-side, i.e. $Kf/(½B_m)$ is nothing other than the restoring moment at the angle ϕ_D, defining the range of initial stability (the moments are divided by g). Inequality (4) shows that the restoring moment at the critical heel angle must be greater than the heeling moment induced by the load at the vessel's side.

3. Inclining test

The coefficient of mass stiffness can be determined precisely with the help of an *inclining test*, which does not require any technical documentation of the vessel. The coefficient does not depend on the displaced mass. Hence, from equation (1) it follows that:

$$K_p = m_p e/\tan\phi, \tag{5}$$

where m_p is the mass used in the inclining test; the mass m_p is small by definition, some several dozen of kilograms, to produce an angle of heel not less than one degree.

Equation (5) determines the stiffness K_p of the vessel together with the mass m_p used in the inclining test. The mass m_p modifies the boat stiffness according to the equation:

$$K_p = K_0 + m_p(z_m - z_p), \tag{6}$$

where K_0 is the vessel's stiffness without the inclining weights, $z_m = T + ½\Delta T + r_C$ is the ordinate of the differential metacentre, ΔT is the increase of immersion (sinkage), z_p is the ordinate of the mass center m_p, whereas $r_C = \Delta J/\Delta V = J'/A_{WL}$ is the metacentric differential radius ($J' \equiv dJ/dz$), i.e. the radius of curvature of the curve of the centre of flotation C, i.e. the geometrical centres of the waterplane; $r_C > 0$ is positive. The actual stiffness of the boat $K_0 < K$ is smaller than the measured stiffness K, provided the gravity centre of m_p is below the differential metacentre. The difference in stiffness drops, if the mass used in the inclining test drops.

The differential metacentric radius can be easily estimated assuming that the coefficient of cross-sections $\beta = const$ is constant relative to draught. It can be easily proven then that $\beta = \kappa$, i.e. the coefficient β is equal to the vertical prismatic coefficient $\kappa = V/A_{WL}T = \delta/\alpha$. Note that $\delta \equiv C_B$, $\alpha \equiv C_W$, and $\kappa \equiv C_{PV}$. An approximation of a cross-section by a power function reads:

$$y = y_0(z/z_0)^n,$$

where y_0 is half width of the section at a draught $z = z_0$, while the exponent $n = 1/\kappa - 1$. The transverse moment of inertia of the waterplane is equal to:

$$J = \int_L \tfrac{2}{3} y^3 dx = \int_L \tfrac{2}{3} y_0^3 (z/z_0)^{3n} dx = (z/z_0)^{3n} \int_L \tfrac{2}{3} y_0^3 dx = J_0 (z/z_0)^{3n},$$

where J_0 is the moment of inertia of the waterplane at a draught $z = z_0$. As we can see, the moment of inertia is also a power function of draught of the exponent $q = 3(1/\kappa - 1)$, that is three times larger than the exponent for frames. Now, it can be easily proven that if a curve of the waterplane inertia is approximated by a power function $J = J_0(z/z_0)^q$, then the differential metacentric radius $r_C = J'/A_{WL}$ at a point $z = z_0$ equals $r_C = q\kappa r_B$, where $r_B = J/V$ is the transverse metacentric radius. As we can see, the differential metacentric radius r_C is proportional to the metacentric radius r_B. Substituting $q = 3(1/\kappa - 1)$, a simple estimate for the differential metacentric radius is obtained:

$$r_C \approx 3(1 - \kappa) r_B. \qquad (7)$$

In the case of fishing boats the vertical prismatic coefficient is relatively small, $\kappa = 0{,}40 \div 0{,}85$ varies in a wide range, with an average value of $\kappa \approx 0.65$. Therefore, the differential metacentric radius r_C can be larger than the metacentric radius r_B, and hence the differential metacentre can lie above the metacentre.

Supplementary data is required to benefit from equation (7), such as the vertical prismatic coefficient κ and the metacentric radius r_B. In this situation a good way out is to apply possibly small heeling weights, marginally changing the stiffness of the boat and resulting in small heeling angles, which is acceptable as long as the heeling angle is precisely measured.

4. Criterion

The key issue in applying relation (4) as the stability criterion is the knowledge of the mass of the nominal weight m, charging the vessel's side. Reference [1] postulates $m = 0{,}13M$, that is 13 percent of the vessel's mass, which is virtually unknown. For the needs of the criterion an estimate of the ship's mass is sufficient:

$$M = \rho LBT\delta = \rho\delta(T/B)LB^2 = \rho c_M LB^2, \qquad (8)$$

where L, B, T are the main particulars of the boat at the waterplane during the inclining test (outside the hull), δ is the block coefficient, ρ is the water density, and $c_M \equiv \delta/(B/T)$ is the coefficient of the vessel's displacement.

For five fishing boats analysed in reference [1], the displacement factors c_M were from the range $0{,}10 \div 0{,}15$, with an average value $c_M \approx 0.12$.. In the case of 35 boats tested by PRS, the displacement factors were from the range $0.07 \div 0.25$, with an average value $c_M \approx 0{,}15$.

As the nominal mass $m = 0.13M$, we obtain:

$$m = c_m \rho LB^2, \qquad (9)$$

where the coefficient of the nominal mass $c_m = 0.13c_M$, while the coefficient of displacement $c_M \equiv \delta/(B/T)$. In the case of lack of data we can assume $c_M = 0.14$. Due to the wide spread of the coefficient of displacement, resulting from a wide spread of the ratio B/T, it is recommended to assess the coefficient of displacement using the said equation. The ratio of B/T for tested boats was from the range $1.75 \div 6$.

Substituting to equation (4) the mass m, given by equation (9), we get the stability criterion in the form:

$$Kf \geq ¼c_m \rho L B^2 B_m^{\,2}. \qquad (10)$$

The above criterion does not require any technical documentation. If no data is available we can adopt the nominal mass coefficient $c_m = 0.13 \cdot 0.14 = 0.0182$, irrespective of the vessel size.

Criterion (10) can be presented in terms of the righting arms. Dividing it by $½MB_m$, where M is the vessel mass, we obtain:

$$l_D \geq l_C, \qquad (11)$$

where $l_D = (K/M)_0 t_0$ is the righting arm at the angle ϕ_D, i.e. at the angle at which the deck edge is immersed or the (assumed) bilge comes out of the water, whichever is smaller, determined on the basis of the measurement of boat stiffness K_0 and freeboard f_0, M is the boat mass, $t_0 \equiv \tan \phi_D = f_0/½B_m$ is the tangent of the angle determining the range of initial stability, and

$$l_C = 0{,}13 \cdot ½B_m = 0.065 B_m$$

has the meaning of the *minimum* righting arm (lower bound), identical with the *maximum* heeling arm. As we can see, it is proportional to the vessel breadth B_m. According to IMO, however, parameters of the righting arm, treated as minimal, remain constant for ships with length $L > 24$ m, which means that for bigger vessels the maximum heeling arm l_C should also be constant.

Criterion (11) generally applies to vessels for which the heeling moment is produced by people and equipment placed on vessel side, as adopted in the assumptions. These are predominantly fishing vessels up to 24 m in length. Bigger vessels are assumed to have technical documentation. For such ships criterion (10) could be treated as an interim criterion.

In criterion (10) the nominal mass m weighing down the boat's side is 13% of the vessel's mass. It follows from this criterion that the minimum righting lever at the angle at which the deck edge is immersed in water equals $l = 0{,}065B_m$. For vessels of breadth $B = 3$ m, this yields $l = 0.195$ m, which seems to be reasonable. It proves that the value of the nominal mass is fixed correctly.

As the right hand-side of criterion (10) grows with the size of the ship it needs to be limited to comply with the IMO philosophy regarding stability criteria. It can be done in two ways. When the boat length is over 20 m, the boat length in criterion (10) should be taken as $L = 20$ m. Alternatively, the minimum restoring arm l_C in criterion (11) need not be bigger than, for example, $l_C = 0.32$ m.

5. Extrapolation

Criterion (10), in terms of moments, and criterion (11), in terms of arms, apply basically to boats in a testing condition, theoretically without cargo. We want to extend these criteria for other loading conditions. There is no problem with freeboard f on the left hand-side of the inequality (10), nor with its right hand-side, it is necessary to take simply data for full loading condition. The problem lies in the stiffness of fully loaded boat, which cannot be determined from the technical documentation because there is no documentation, nor can it be determined from the inclining test, as the vessel is usually unloaded during the test. Thus, there is a need to estimate it somehow. One of the options is the introduction of the stiffness ratio: $c_K \equiv K_c/K_0$ that depends virtually on the draught ratio. This relationship could be established analysing data of boats with technical documentation.

The stiffness of a loaded boat is given by equation (6):

$$K_c = K_0 + m_c(z_m - z_c), \qquad (12)$$

where m_c is the mass of cargo, $K_0 \equiv M_0 h_0$ is the boat stiffness from the inclining test for the boat in the test condition, z_c is the ordinate of the cargo centre of gravity, $z_m = T + ½\Delta T + r_C$ is the ordinate of the differential metacentre, and ΔT is the sinkage. If we assumed that the differential metacentric height $h_m \equiv z_m - z_c$ is approximately the same as the metacentric height h_0, then the stiffness K_c would be approximately proportional to the increase of the vessel's mass. In this case the stiffness ratio $K_c/K_0 \approx M_c/M_0$ would be equal to the displacement ratio. In effect, $c_K \approx M_c/M_0 = V_c/V_0 = 1 + \Delta V/V_0$. As generally, $h_m \neq h_0$, i.e. the differential metacentric height is not equal to the initial metacentric height, a stiffness-correcting coefficient could read:

$$c_K = 1 + c_1 \frac{\Delta V}{V_0} = 1 + c_1\left(\frac{V_c}{V_0} - 1\right), \tag{13}$$

where c_1 is the coefficient to be determined using data of known boats.

There is nothing to prevent us from calculating the stiffness K_c using equation (12). Assuming that the ordinate of cargo gravity centre $z_c = H$, we obtain:

$$K_c = K_0 + m_c[T_s + 3(1 - \kappa)r_B - H)],$$

where the mass of cargo $m_c = M_c - M_0$, $T_s = ½(T_0 + T_c)$ is the average draft, and r_B is the average metacentric radius. Dividing the above equation by K_0, we obtain:

$$c_K = 1 + \frac{m_c}{M_0}\frac{M_0}{K_0}\left[T_s + 3(1-\kappa)r_B - H\right].$$

Accounting that $m_c/M_0 = \Delta V/V_0 = V_c/V_0 - 1$, we obtain:

$$c_K = 1 + \frac{M_0\left[T_s + 3(1-\kappa)r_B - H\right]}{K_0}\left(\frac{V_c}{V_0} - 1\right).$$

By comparing with equation (13) we can see that the coefficient c_1 reads:

$$c_1 = \frac{T_s + 3(1-\kappa)r_B - H}{h_0} = \frac{r_C - f_s}{GM_0}, \tag{14}$$

where $r_C = 3(1 - \kappa)r_B$ is the average differential metacentric radius, r_B is the transverse metacentric radius, $f_s \equiv H - T_s$ is the average freeboard, $h_0 \equiv K_0/M_0$ is the initial metacentric height GM_0 in the test condition, the stiffness K_0 is from the inclining test, and M_0 is the vessel's mass during the test, estimated using the general equation (8), to which we substitute values corresponding to the test condition. The knowledge of the coefficient c_1 allows for the determination of the stiffness ratio $c_K = K_c/K_0$, which in turn allows for the determination of the stiffness K_c for the boat with cargo. Thus, criterion (10) assumes the following form:

$$K_c f_c \geq ¼ c_m \rho L B^2 B_m^2. \tag{15}$$

In the case of criterion (11), the right hand-side does not change but the left hand-side assumes the following form:

$$(K_c/M_c)t_c \geq l_C. \tag{16}$$

where $K_c = c_K K_0$, and M_c can be calculated from the general formula (8), $t_c \equiv \tan\phi_D = f_c/½B_m$, while $f_c = f_0 - \Delta T$ is the freeboard in loaded condition, equal to the freeboard in the test condition, reduced by the sinkage ΔT, induced by cargo, where $\Delta T/T_0 = (V_c/V_0)^\kappa - 1$, while $\kappa \equiv \delta/\alpha$ is the vertical prismatic coefficient.

6. Non-linear model

Stability criteria (15) and (16) are based on the metacentric equation (2), which is a linear function of $\tan\phi$, and same – a linear function of freeboard f. This criterion could be refined by accounting for the nonlinearity of the GZ-curve in the range of initial stability. Equation (2) would take then the form:

$$D(h_0 + ½ r_B \tan^2\phi)\tan\phi \geq pe.$$

The left hand-side is correct for wall-sided vessels. For vessels with flared frames, the left hand-side is conservative, that is, in reality it is somewhat higher. Dividing it by acceleration due to gravity g and substituting $e = ½B_m$ we obtain a stability criterion in a non-linear format:

$$Kt + ½\rho J t^3 \geq ½ m B_m,$$

where J is the waterplane inertia moment, and $t \equiv \tan\phi$. Applying analogical substitutions as before, the criterion for a fully loaded condition takes the form:

$$K_c t_c + ½\rho J_c t_c^3 \geq ½ c_m \rho L B^2 B_m, \tag{17}$$

where $t_c = f_c/(½B_m)$ is the tangent of the angle determining the range of initial stability, $K_0 = M_0 GM_0$ is the vessel's stiffness in the test condition, $K_c = M_c GM_c$ is the stiffness in loaded condition, whereas f_c and J_c are the freeboard and waterplane inertia moment in loaded condition at the deepest draught. The stiffness $K_c = c_K K_0$, where the coefficient c_K is defined by equation (13), while c_1 – by equation (14).

If there is no data, the waterplane inertia moment can be calculated using the equation:

$$J_x \approx c_x L B^3, \tag{18}$$

where c_x is the coefficient dependent on the waterplane coefficient of fineness. Approximation of the waterline quarter by a power function is of the form: $y = ½B[1 - (x/x_0)^n]$, where $x_0 = ½L$ is half the waterline length, exponent $n = \alpha/(1 - \alpha)$, and α is the waterplane coefficient of fineness. The coefficient c_x for such a waterline is equal to:

$$c_x = \frac{\alpha^3}{2(1+\alpha)(1+2\alpha)}. \tag{19}$$

For a rectangle, $\alpha = 1$, the above equation yields $c_x = {}^1/_{12}$, for a rhombus (the bow and stern parts are triangles), $\alpha = ½$, $c_x = {}^1/_{48}$, as it should be. Usually, the angle of sharpness of the waterline is smaller than for parabolic waterlines. Therefore, the real coefficient c_x is slightly higher than for parabolic ones. This coefficient is also higher for waterplanes with a finite width at the stern. In such cases, the coefficient c_x should be calculated according to the Appendix.

The metacentric radius $r_B = J/V$ can be calculated from the equation:

$$r_B = (c_x/c_M)B, \tag{20}$$

where $c_M \equiv \delta/(B/T)$ is the vessel's displacement coefficient.

If both sides of inequality (17) are multiplied by $½B_m$, we obtain the equation:

$$K_c f_c + ½\rho J_c f_c t_c^2 \geq ¼ c_m \rho L B^2 B_m^2, \tag{21}$$

In a format similar to criterion (15). The difference lies in the second term on the left hand-side. If both sides of inequality (21) are divided by the vessel's mass M_c, we obtain the stability criterion in terms of righting arms:

$$(K_c/M_c)t_c + ½(r_B)_c t_c^3 \geq l_C, \tag{22}$$

where r_B is the transverse metacentric radius. The format of the criterion is identical to criterion (16); the difference lies in the second term on the left hand-side. If no data is available we can apply analytical formulations, well approximating real values, such as equations (18) and (20).

For boats with flared frames the metacentric radius in the initial range of stability $r \approx const$ is practically constant (Fig. 1). In effect the metacentre is constant, reducing the righting arm at the initial range of stability to the metacentric formula $l = h_0 \sin\phi$. Consequently, the stability criterion is reduced to the linear one (16).

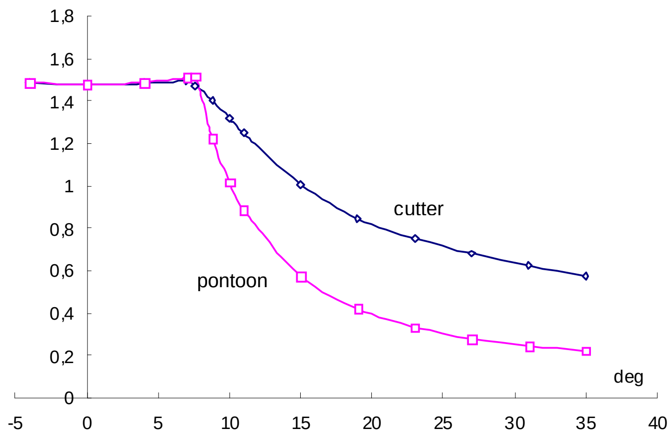

Fig. 1. Exemplary curve of the metacentric radius (WŁA-84)

7. Boats with deck

The above considerations are fully adequate for open boats without a deck. Among the investigated boats there were six such vessels, which all satisfied criterion (16) with ample margin. However, in the case of boats with deck, nearly half of the boats (precisely 37 out of 58) did not satisfy the criterion, though they had decent *GZ*-curves.

For open boats the righting arm l_D at the angle of deck edge immersion is the maximum arm. In the case of boats with a deck the deck edge may be immersed in water. Thus, the maximum righting arm occurs for angles beyond the range of initial stability. For such boats the stability criterion remains the same provided that the arm l_D is treated as the maximum righting arm l_{max}.

The determination of the maximum righting arm is much more involved than determination of the righting arm l_D at the angle of deck edge immersion or the (assumed) bilge emergence. For the angle of heel $\phi = \phi_{max}$, where the *GZ*-curve reaches maximum, the derivative of the *GZ*-curve, which is the same as the metacentric height $h = r - BZ$, vanishes, where $BZ = a + l_d$ is the height of the centre of gravity above the centre of buoyancy, $a = z_G - z_B$ is the height of the centre of gravity above the centre of buoyancy in the upright position, and l_d is the dynamic arm, i.e. the integral curve of the righting arm l. Thus, the angle ϕ_{max} determines the equation: $r = BZ$. Hence,

$$r = a + l_d, \tag{23}$$

where $a \equiv z_G - z_B = r_0 - h_0$ is constant, easy to determine. The dynamic lever equals:

$$l_d \approx (l_d)_D + l_D\Delta\phi + ⅔\Delta l\Delta\phi, \tag{24}$$

where l_D and $(l_d)_D$ are the righting and dynamic arms at the angle of deck edge immersion ϕ_D, respectively, while $\Delta\phi \equiv \phi_{max} - \phi_D$, and $\Delta l \equiv l_{max} - l_D$ is the increase of the GZ-curve; the constant ⅔ results from adopting a square approximation for the GZ-curve between the angle ϕ_D and ϕ_{max}.

With the square approximation of the GZ-curve after exceeding the angle ϕ_D the maximum righting lever is equal to $l_{max} = l_D + ½l_D'\Delta\phi$, where $l_D' \equiv h_D$ is the derivative of the GZ-curve at the angle ϕ_D. Assuming that at the range of initial stability the GZ-curve is expressed by the metacentric equation $l = h_0\sin\phi$, thus $h_D = h_0\cos\phi_D$, and the increase of the GZ-curve equals:

$$\Delta l \equiv ½h_0\Delta\phi\cos\phi_D.$$

By accounting for this result the last term in equation (24) reads:

$$⅓h_0(\Delta\phi)^2\cos\phi_D,$$

whereas the right hand-side of equation (23), i.e. the segment BZ, reads:

$$BZ = r_0 - h_0\cos\phi_D[1 - t_c\Delta\phi - ⅓(\Delta\phi)^2], \qquad (25)$$

where $t_c = f_c/(½B_m)$. Should we assume that the run of the metacentric radii above the angle ϕ_D is analogical to that for a rectangular pontoon, then

$$r = \frac{r_0}{\cos^3\phi}\left(\frac{t_c}{\mathrm{tg}\,\phi}\right)^{1,5} = r_0\left(\frac{t_c}{\cos^2\phi\,\mathrm{tg}\,\phi}\right)^{1,5} = r_0\left(\frac{t_c}{\cos\phi\sin\phi}\right)^{1,5}.$$

Considering that $\sin\phi\cos\phi = ½\sin2\phi$, we get

$$r = r_0\left(\frac{2t_c}{\sin 2\phi}\right)^{1,5} \qquad (26)$$

where r_0 is the initial metacentric radius. As can be seen from Fig. 1, metacentric radii for boats are larger then indicated by equation (26). Introducing a parameter increasing the metacentric radii in relation to equation (26), denoted by s, we obtain:

$$r = sr_0\left(\frac{2t_c}{\sin 2\phi}\right)^{1,5},$$

In general, the factor s varies with the angle of heel. For the sake of simplicity a constant value of s can be assumed, equal to the ratio of metacentric radii in an upright position. Using equation (20) for the initial metacentric radius the following is obtained for the factor s:

$$s = 12c_x/\delta, \qquad (27)$$

where $s \leq 0{,}9s_{max}$ should not be greater than a certain maximum value, which will be discussed later. This is a rough approximation for the average s value.

Taking into account the foregoing considerations equation (23) reads:

$$sr_0(2t_c/\sin2\phi)^{1,5} = r_0 - h_0\cos\phi_D[1 - t_c\Delta\phi - ⅓(\Delta\phi)^2].$$

Dividing both sides by the initial metacentric radius r_0 we get a non-dimensional form:

$$s(2t_c/\sin2\phi)^{1,5} = 1 - (h_0/r_0)\cos\phi_D[1 - t_c\Delta\phi - ⅓(\Delta\phi)^2], \qquad (28)$$

where $\Delta\phi \equiv \phi_{max} - \phi_D$ is in radians. This is a transcendental equation relative to $\phi \equiv \phi_{max}$, which can be solved iteratively, e.g. using „Goal seek” in Excel. The equation may have no solution if a graph of the left hand-side exceeds a maximum value of the right hand-side, i.e. if the factor s is too big, that is to say, if $s > s_{max}$. The accuracy of solution for ϕ_{max} depends on the accuracy of estimation of the factor s.

Equation (28) determines $s = s(\phi)$ as a function of the angle of heel (Fig. 2); the factor s is understood as the ratio of the function on the right hand-side and the power function on the left hand-side of equations (28). A maximum value s_{max} corresponds to the maximum of the function $s = s(\phi)$, which for the investigated boats occurs almost exactly at the angle $\phi = 50°$, irrespective of the parameters t_c and h_0/r_0. However, if the parameter h_0/r_0 is over-predicted maximum of the function s can occur at angles higher than 50°. In such cases s_{max} should be taken for the angle 50°. The maximum drops rapidly with the rise of the parameter $t_c \equiv \tan\phi_D$, i.e. with the rise of the range of initial stability. For the range of interest, it is marginally affected by the parameter h_0/r_0.

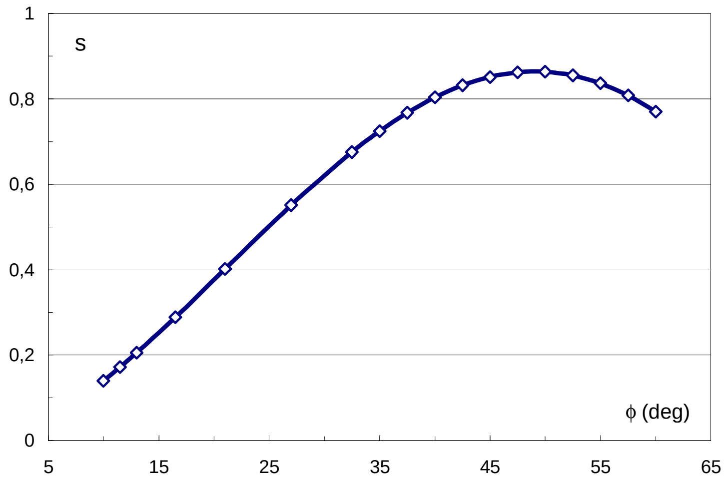

Fig. 2. Factor s

The knowledge of ϕ_{max} allows for determination of the ratio l_{max}/l_D, i.e. the degree of an increase of the maximum arm relative to l_D. The maximum lever $l_{max} = l_D + \Delta l$, thus

$$l_{max} = l_D + ½h_0\Delta\phi\cos\phi_D.$$

Dividing both sides of the equation by $l_D = h_0\sin\phi_D$ and introducing the notation $c_D \equiv l_{max}/l_D$, the impact of the deck on the increase of the maximum lever takes the form:

$$c_D = 1 + ½\Delta\phi/t_c, \qquad (29)$$

where $\Delta\phi \equiv \phi_{max} - \phi_D$, and ϕ_{max} is the root of equation (28). With the help of the factor c_D the maximum lever is equal to $l_{max} \equiv c_D l_D$, the increase of the maximum lever $\Delta l = (c_D - 1)l_D$, and $\Delta\phi = 2(c_D - 1)t_c$.

Determination of the factor c_D, as above, refers to the situation when the angle of flooding ϕ_f is greater than the angle ϕ_{max} at which the GZ-curve reaches maximum. Otherwise, the factor c_D is affected by the angle of flooding, as follows:

$$c_D = 1 + ½c_f\Delta\phi/t_c,$$

where $c_f = 1 - (\Delta\phi_f/\Delta\phi)^2$ is a reduction coefficient, accounting for the angle of flooding ϕ_f, if it is smaller than the angle ϕ_{max}, while $\Delta\phi_f \equiv \phi_f - \phi_D$. Determination of the angle of flooding without the technical documentation is a quite difficult task. It can be visually estimated, in proportion to the angle of deck edge immersion. The quantity $\Delta\phi \equiv \phi_{max} - \phi_D$ remains unaffected by the angle of flooding.

Fig. 3 presents the correlation between $l_{max} = c_D l_D$, where c_D is given by equation (29), with the angle ϕ_{max} as the root of equation (28), and the actual l_{max}, taken from the vessel's documentation. The levers are in meters. As can be seen, on average l_{max} is overestimated by 16.5 %.

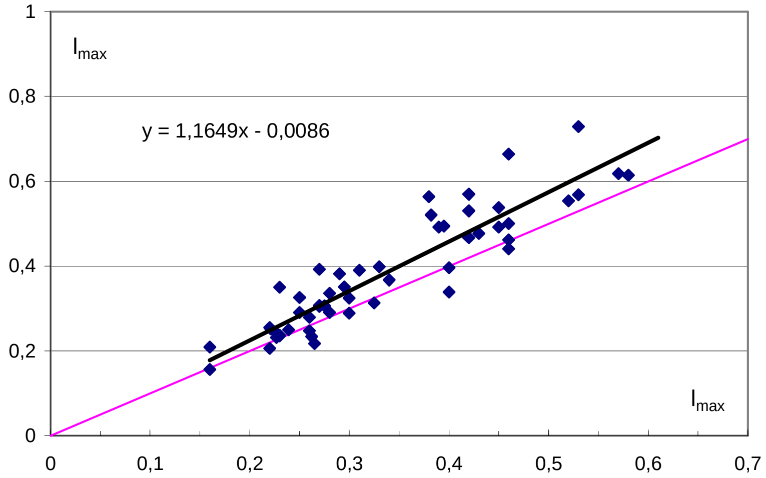

Fig. 3. Correlation between the estimated and actual l_{max} values

If ϕ_{max} in equation (29) is taken from the vessel's documentation we get the correlation presented in Fig. 4. Now, the value l_{max} is overestimated by 10,7 %. If the data are correct, this could be the effect of the square approximation of the *GZ*-curve in the left hand-side proximity of the peak. The spread of points, similarly as in Fig. 3, results from a rough approximation of the metacentric radii for large heel angles $\phi > \phi_D$, as well as from the occurrence of deck sheer and superstructure. The spread is also affected by the assumed nature of the bilge, when the freeboard is big. In view of the above remarks, the correlation between the calculated and real points appears to be quite good. The correlation depends on the quality of estimation of the factor *s*. Equation (27) is only a rough approximation of this factor.

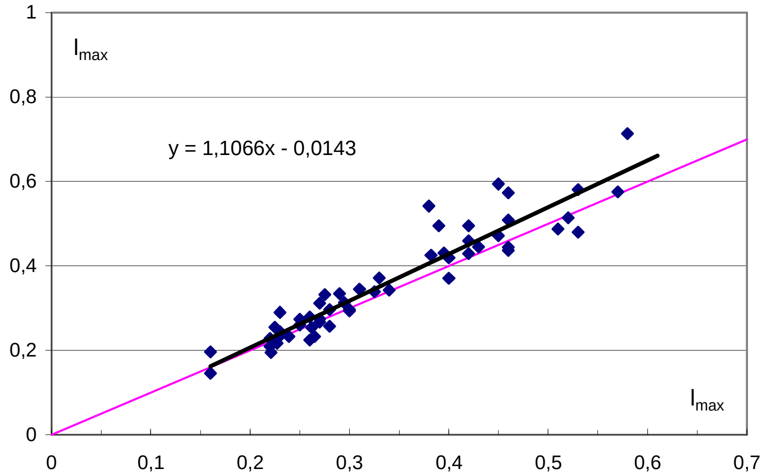

Fig. 4. Correlation between the estimated and actual l_{max} values if ϕ_{max} is taken from documentation

The criterion for boats with deck would take the form:

$$l_{max} > l_C, \tag{30}$$

where $l_{max} \equiv c_D l_D$. Strictly speaking, the static heeling arm changes with the angle of heel ϕ as $\cos\phi$. In such a situation, the stability criterion would take the form:

$$l_{max} > l_C \cos\phi_{max}.$$

Neglecting $\cos\phi$, i.e. assuming a constant value for the heeling arm l_C makes criterion (30) more stringent, leaving room for some *dynamic* effects, unavoidable in operation of fishing boats.

8. Estimation of the coefficients of hull fineness

As we can see stability assessment depends on the main particulars of the vessel and on two coefficients of hull fineness: the waterplane coefficient α, and the block coefficient δ. The two coefficients determine the vertical prismatic coefficient κ, as $\kappa = \delta/\alpha$. The main particulars of the vessel can be easily established by direct measurements. Regarding the two coefficients of fineness, they can be estimated with the help of a sinkage test.

The sinkage test is carried out for the boat in the test condition, for which the stiffness K_0 and freeboard f_0 were measured. A weight of mass m_1 is next taken on board the vessel, which changes displacement by $\rho\Delta V$. Change of displacement must be equal to change of the vessel mass, i.e. the mass of the weight. Thus, $m_1 = \rho\Delta V$, where ρ is the density of water.

If the adopted weight lies on the vertical line passing through the centre of flotation, i.e. the centre of gravity of the waterplane, the resultant sinkage is parallel. Therefore, change in displacement reads $\Delta V = A_{WL}\Delta T$. Thus, $m_1 = \rho A_{WL}\Delta T$, which defines the waterplane area:

$$A_{WL} = m_1/\rho\Delta T. \tag{31}$$

The above equation includes division of two small quantities: m_1 and ΔT. For the result to be robust the weight must be such that the sinkage is not less than 3÷4 cm. Change of immersion $\Delta T = -\Delta f = f_0 - f_1$ equals change of freeboard with the opposite sign. Change of immersion can also be measured with the help of a water pipe, provided it is connected to outboard water below the waterline.

The knowledge of the waterplane area defines the coefficient of waterplane:

$$\alpha = A_{WL}/LB, \tag{32}$$

but not less than 0,60. In the next step the inclining tests could be performed, yielding the mass stiffness K_1 for the vessel with the additional mass m_1. The stiffness K_1 for the boat with the mass m_1 is given by the equation:

$$K_1 = K_0 + m_1(T - z_1 + ½\Delta T + r_C), \tag{33}$$

where z_1 is the ordinate of the centre of gravity of the adopted weight, and $r_C = 3(1 - \kappa)r_B$ is the differential metacentric radius for a layer of additional displacement. The value $T - z_1$ is the distance of the centre of gravity of the adopted weight from the initial waterplane; $T - z_1 > 0$ is positive, when the centre of gravity of the adopted weight lies below the waterline. The only unknown term in equation (33) is the differential metacentric radius r_C, which this equation defines. The knowledge of r_C, however, does not help in determining the displacement of the vessel, as the block coefficient δ remains unknown. Thus, we must determine this coefficient from a regression formulation:

$$\delta = 0{,}775\alpha - 0{,}131, \tag{34}$$

but not less than 0,30.

9. Proposed criterion

Taking into account the foregoing considerations, a stability criterion is suggested in terms of righting levers. A vessel is deemed to meet stability requirements if it satisfies the inequality:

$$c_D l_D \geq l_C, \tag{35}$$

where c_D reflects the impact of deck on the increase of the maximum righting arm, l_D is the righting lever, the same as in equation (16), at the angle ϕ_D determining the range of initial stability, and $l_C = 0{,}065 B_m$, (but not more than 0.32 m) is a minimum righting lever. For open deck boats the factor $c_D = 1$, whereas for boats with deck, it is given by equation (29).

Out of 58 examined vessels 16 do not meet the above criterion. Should we apply in the criterion the angle ϕ_{max} given in the documentation of vessels, 18 vessels would fail. Those vessels that fail, have *GZ*-curves with minimum parameters, such as l_{max} in the proximity of 0.20 m, or ϕ_{max} in the proximity of 25°.

If the right-hand side of criterion (35) is multiplied by $\cos\phi_{max}$, then 9 vessels would fail such a criterion. It is proposed, therefore to keep the criterion, as it is, given by equation (35).

References

1. Frąckowiak M., Pawłowski M. *The safety of small open deck fishing boats*, Proceedings, 3rd Int. Conference on Stability of Ships and Ocean Vehicles, Gdansk 1986, Vol. 2, pp. 201–207, Gdansk, September 1986.
2. International Maritime Organisation, *Intact stability criteria for passenger and cargo ships*, IMO, London 1987, sales number IMO–832E, 22 s.
3. Deakin, B. *Spend Less, Save More (Lives)*, Proceedings, 11th Int. Conference on the Stability of Ships and Ocean Vehicles, September 2012, Athens, Greece, pp. 689–697.

Appendix – Moment of inertia for a transom waterplane

We want to derive an expression for the transverse moment of inertia for a waterplane described by the waterplane coefficient of fineness α for the whole waterplane and the width *b* of the transom at stern (Fig. 5).

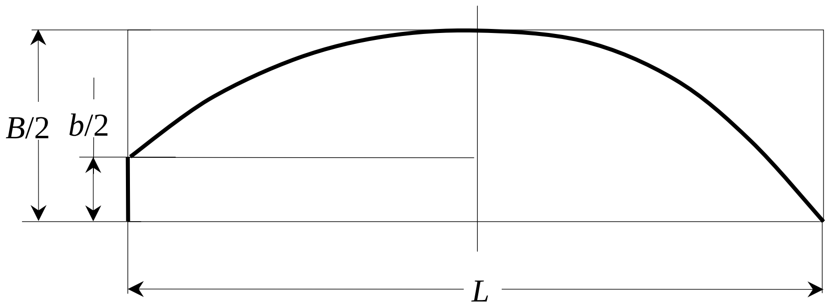

Fig. 5

The area of the waterplane is given by the equation:

$$LB\alpha = \tfrac{1}{2}Lb + \tfrac{1}{2}L(B - b)\alpha_A + \tfrac{1}{2}LB\alpha_F,$$

where α_A and α_F are the coefficients of fineness for the waterline parabola in the aft and forward parts of the vessel. Dividing this equation by ½*LB*, we get:

$$2\alpha - b' = (1 - b')\alpha_A + \alpha_F,$$

where $b' = b/B$ is a relative width of the transom. It can be assumed that the coefficients of fineness $\alpha_A = \alpha_F \equiv \alpha'$ for the aft and forward parts of the waterline are the same and equal to a modified coefficient of fineness α'. From the above equation we get immediately:

$$\alpha' = (2\alpha - b')/(2 - b'). \qquad (36)$$

The quantities α i b' are given from the measurements. The same coefficients of fineness for the aft and forward segments of the waterline mean that the two segments are affine.

A parabolic waterline is given by the equation: $y = ½B[1 - s'(x/x_0)^n]$, where $x_0 = ½L$ is half-length of the waterplane. The exponent $n = \alpha'/(1 - \alpha')$, whereas $s' = 1 - b'$ is a relative sagitta of a parabolic arc.

The transverse moment of inertia for either part of the waterplane is given by the equation:

$$J_x = ⅔\int_0^{x_0} y^3 dx = ⅔(½B)^3 \int_0^{x_0} [1 - 3s'(x/x_0)^n + 3s'^2(x/x_0)^{2n} - s'^3(x/x_0)^{3n}]dx.$$

For the forward part $s' = 1$. Considering that the integral of a monomial $(x/x_0)^p$ is equal to:

$$\int_0^{x_0} (x/x_0)^n = x_0/(p + 1),$$

it easy to get the expression for the moment of inertia for one half of the waterplane:

$$J_x = ⅔(½B)^3 x_0 [1 - 3s'/(n + 1) + 3s'^2/(2n + 1) - s'^3/(3n + 1)].$$

Substituting $x_0 = ½L$, the following is obtained:

$$J_x = {}^1/_{24}LB^3[1 - 3s'/(n + 1) + 3s'^2/(2n + 1) - s'^3/(3n + 1)],$$

where $n = \alpha'/(1 - \alpha')$, the same for the aft and forward part. If the expression in the brackets be denoted by c_A for the aft part, and c_F for the forward, the expression for the inertia moment is reduced then to equation (18), in which the coefficient $c_x = (c_A + c_F)/24$.

When waterline dimensioning is accomplished, the area and transverse moment of inertia of the waterplane can be obtained with the help of numerical integration. Typically, 5–6 ordinates will be measured. For the purpose of calculation the waterline can be approximated by a broken line, which entails the trapezoidal rule. The integration is as follows:

$$A_{WL} = 2\int_0^L y dx = \Sigma\Delta x_i(y_{i-1} + y_i),$$

$$J_x = ⅔\int_0^L y^3 dx = ⅓\Sigma\Delta x_i(y^3_{i-1} + y^3_i),$$

where the summation refers to the sub-intervals. Since lengths of sub-intervals are generally different, the length of a sub-interval Δx_i cannot be taken in front of the summation sign.

Estimation of the block coefficient δ is based on the waterplane coefficient of fineness $\alpha = A_{WL}/LB$. The block coefficient δ is obtained from equation (34). This equation applies basically to vessels without a transom. In the case of a transom waterplane, the waterplane coefficient of fineness α in equation (34) should be replaced by the coefficient α', given by equation (36).